

Probabilistic Assessment of the Structural Reliability of Reinforced Concrete Beams

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Abstract. The paper presents the development of theoretical and applied principles for the probabilistic assessment of the reliability of reinforced concrete structures. A method for evaluating structural reliability at the construction stage is proposed, based on a probabilistic approach and statistical processing of factors influencing load-bearing capacity. Analytical relationships for determining the probability of failure-free operation using the reliability index are obtained, and a comparison between deterministic and probabilistic approaches is carried out. It is established that the use of probabilistic methods contributes to an increase in the reliability of structures. The results obtained may be applied in assessing the reliability of building structures at the design stage of construction projects.

Keywords: reinforced concrete structures, flexural beam, reliability, probability of failure-free performance, reliability index, probabilistic calculation method, statistical linearization, reinforcement, load-bearing capacity, design.

Introduction

Based on the analysis of scientific publications, it was found that existing approaches do not sufficiently account for the uncertainty of operational parameters and require development of interdisciplinary methods.

Armen Der Kiureghian [1] specializes in reliability analysis using Bayesian methods, finite element modeling, and reliability-based optimization, which has significantly advanced the applied aspects of structural assessment.

Liu Haobo [2] developed calculation models that take into account the uncertainty of building structure parameters, enabling more accurate predictions of their behavior under various operating conditions.

S. Soloviev [3] proposed a P-box-based approach for reliability analysis under uncertainty, enabling risk assessment without classical

probability assumptions.

Among Kazakhstan scientists, D.A. Akhmetov [4] studied methods of concrete strength testing used in structural inspections, comparing destructive and non-destructive approaches.

In [5], the design and construction of buildings using precast concrete blocks, the efficiency of construction processes, and structural reliability were examined.

In [6], a methodology for assessing structural reliability under incomplete probabilistic information was developed based on Bayes' law and modal analysis, demonstrating higher efficiency than the Gibbs sampling method.

In [7], it is noted that deterministic optimization does not explicitly account for existing uncertainty.

In design codes, uncertainties are consid-

ered through safety factors; however, deterministic approaches reduce the calculated reliability level. Therefore, probabilistic methods and the analysis of random variables are more effective. Despite existing studies, many approaches remain limited, and there is a need for integrated probabilistic methods for reliability assessment under uncertainty.

Research methods

Reliable operation of structures requires accurate models of load-bearing capacity and the absence of limit states throughout their service life [8-10]. The main reliability criteria include the probability of failure-free operation and service life prediction. Therefore, along with deterministic approaches, probabilistic methods should be used, considering parameters as random variables. The paper examines the reliability of reinforced concrete beams at the design stage, including deterministic calculation principles [11].

The design scheme of a single-span beam is shown below in Figure 1.

1. Deterministic calculations

According to Figure 2 (using the values of the parameters x and x_R), the deformation characteristics of the cross-section (with one-sided reinforcement) were calculated using the following data [13]:

a) heavy concrete class – B20 (with a service condition coefficient of $\gamma = 0.9$): ($R_b = 10.5 \text{ MPa} = 107 \text{ kgf/cm}^2$) – design resistance for the limit states of the first group;

b) Working reinforcement along the lower chord – AIII: ($R_s = 365 \text{ MPa} = 3750 \text{ kgf/cm}^2$) – design resistance of the longitudinal reinforcement in tension; ($R_{sc} = 365 \text{ MPa} = 3750 \text{ kgf/cm}^2$) – the same, for compression; ($R_{sw} = 290 \text{ MPa} = 3000 \text{ kgf/cm}^2$) – the same, for transverse reinforcement (stirrups and bent bars).

The rectangular cross-section ($b \times h = 40 \times 60 \text{ cm}$); the beam span $l = 6 \text{ m}$.

The uniformly distributed transverse load: ($q_n = 670 \text{ kg/m} = 0.67 \times 10^{-2} \text{ MN/m}$) is the constant load intensity; ($V_n = 1200 \text{ kg/m} = 1.2 \times 10^{-2} \text{ MN/m}$) is the temporary load intensity.

The strength conditions for a normal cross-section of a reinforced concrete structure are as follows [11, 12]:

$$M_{\max} \leq M_{\text{uct}} = R_s \cdot A_s (h_0 - 0,5x). \quad (1)$$

Let's calculate the maximum moment from the combined action of the constant and temporary loads (γ_f is the overload factor):

$$F_M = M_{\max} = \frac{(q_n \gamma_f + \nu_n \gamma_f) l^2}{8}. \quad (2)$$

To verify the deterministic calculation, a numerical experiment was performed in ANSYS, and the results of displacements and stresses were obtained (Figures 3-4).

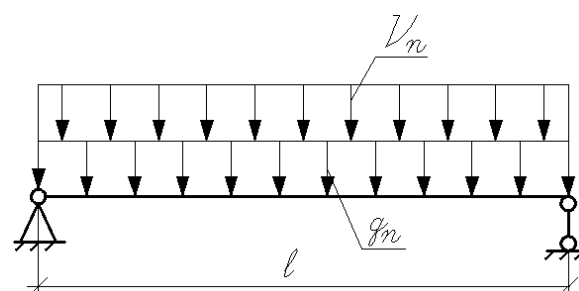


Figure 1 – The design scheme of a beam

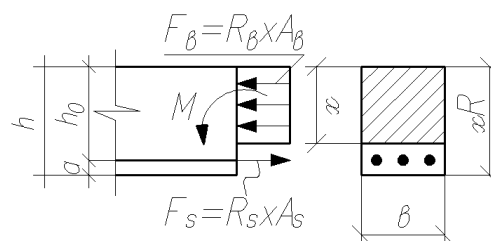


Figure 2 – Towards the issue of the bearing capacity of a reinforced concrete beam

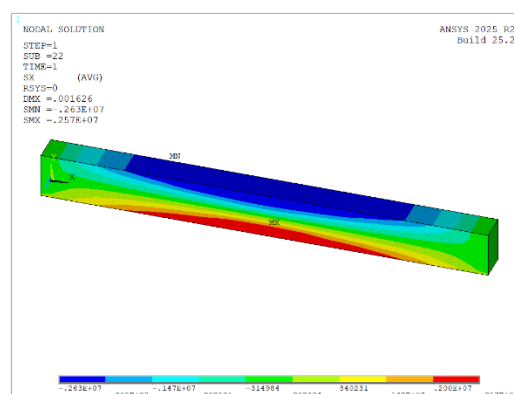


Figure 3 – Normal stresses in the concrete

The obtained results showed satisfactory agreement with the theoretical values, with a relative error of about 14%. The majority of stresses were concentrated in the concrete section, confirming the adequacy of the adopted calculation model.

2. Probabilistic calculations

Write the safety factor function as the probability of failure-free operation (FFO), i.e. probability of fulfilling the strength condition [9]:

$$P_s = p(q > 0); \quad q = F_s - F_M = \tilde{R}_s A_s \cdot \left(\tilde{h}_0 - \frac{\tilde{R}_s A_s}{\tilde{R}_b b} \right) - \left(\frac{\tilde{q}_n \gamma_f + \tilde{\nu}_n \gamma_f}{8} \right) l^2. \quad (3)$$

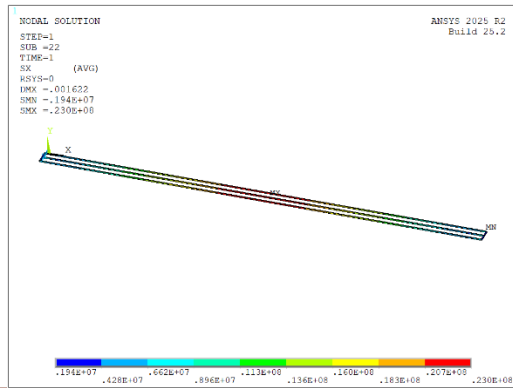


Figure 4 – Normal stresses in the reinforcement

When using probabilistic calculations to assess the reliability level of a structure using random (statistically variable) variables, we use the following parameters (marked with a wavy line).

For the random variables listed above, the following values are specified (based on statistical processing of experimental or design data):

a) Average values:

$$\bar{q}_n = 0,67 \cdot 10^{-2} \text{ MN/m} = 670 \text{ kg/m};$$

$$\bar{R}_s = R_{s,n} = 365 \text{ MPa} = 3750 \text{ kg/cm}^2;$$

$$\bar{l} = l = 6 \text{ m} = 600 \text{ cm};$$

$$\bar{R}_b = R_{b,n} = 10,5 \text{ MPa} = 107 \text{ kg/cm}^2;$$

$$\bar{v} = v_n = 1,2 \times 10^{-2} \text{ MN/m} = 1200 \text{ kg/m};$$

$$\bar{h}_0 = h_0 = 56 \text{ cm} = 0,56 \text{ m};$$

$$\bar{A}_s = A_{s,n} = 0,0036 \text{ m}^2; \quad \bar{b} = 0,4 \text{ m}.$$

Standard deviation:

$$S(R_s) = 18 \text{ MPa} = 180 \text{ kg/cm}^2;$$

$$S(R_b) = 0,54 \text{ MPa} = 5,4 \text{ kg/cm}^2;$$

$$S(h_0) = 0,03v = 0,3 \text{ cm};$$

$$S(l) = 0,3 \text{ m} = 30 \text{ cm};$$

$$S(q_n) = 0,033 \times 10^{-2} \text{ MN/m};$$

$$S(v_n) = 0,06 \text{ MN/m}.$$

According to expression (3), calculate the average value of the bearing capacity reserve:

$$\bar{q}_1 = 365 \cdot 0,036 \left(0,56 - 0,5 \frac{365 \cdot 0,0036}{10,5 \cdot 0,4} \right) - \left(\frac{0,67 \cdot 10^{-2} + 1,2 \cdot 10^{-2}}{8} \right) = 0,4462 \text{ MNm}. \quad (4)$$

Based on the method of statistic linearization [9, 13], calculate variance of the bearing capacity reserve of the structure under consideration:

$$S_{q_1}^2 = S^2(R_{s,n}) \left(\frac{\partial q_1}{\partial R_{s,n}} \right)^2 + S^2(R_{b,n}) \left(\frac{\partial q_1}{\partial R_{b,n}} \right)^2 + S^2(q_n) \left(\frac{\partial q_1}{\partial q_n} \right)^2 + S^2(v_n) \left(\frac{\partial q_1}{\partial v_n} \right)^2 + S^2(h_0) \left(\frac{\partial q_1}{\partial h_0} \right)^2 + S^2(l) \left(\frac{\partial q_1}{\partial l} \right)^2. \quad (5)$$

The required derivatives were determined based on standard statistical linearization.

Substitute the calculated values and the given values (4) into expression (5) and calculate the variance ($S_{q_1}^2$) and standard deviation (S_{q_1}):

$$S_{q_1}^2 = 0,002015; \quad S_{q_1} = \sqrt{S_{q_1}^2} = 0,04489 \text{ MNm}. \quad (6)$$

Calculate the reliability index (taking into account the values of 4, 6):

$$\beta_1 = \frac{q_1}{S_{q_1}} = \frac{0,4462}{0,04489} = 9,94. \quad (7)$$

Calculate the probability of failure-free operation (FFO) of the structure (Figure 1) (according to Table A.5 [10]):

$$P_{s1} = 1 - \Phi(\beta_1) = 1 \cdot 10^{-10} = 0,999(9), \quad (8)$$

where $\Phi(\beta_1)$ is the Gauss function.

According to value (8), the probability of failure-free operation for the reinforced concrete beam in Figure 1 is very high (close to 100%).

Scientific results

Studying the probability of failure-free operation taking into account the reinforcement percentage.

It should be noted that the previous calculations were performed with the beam cross-section conventionally reinforced with class AIII reinforcement with a conventional reinforcement percentage of $\mu = 1.5\%$ (with the reinforcement area of $[A_s = (bh)\mu = 36 \text{ cm}^2]$). Now perform the following study: by varying the value of μ within the range of (0.2%) to (0.25%), determine the required reinforcement area for a given failure-free operation value of greater than 98%. The calculations are presented in tabular form (Table).

Based on the data of Table, calculate the mathematical expectation (average value) of the bearing capacity reserve: $m_q = 0,016923 \text{ MNm}$.

According to formulas XIX.1 (p. 361) [14]: $\xi_R = 0,69$; $\xi_0 = 0,8416$; accept $(\xi = 0,4) < (\xi_R = 0,69)$:

$$F_a = \xi b h_0 \frac{R_{wp}}{R_n} \Rightarrow F_a = 4,19 \text{ cm}^2;$$

$$F_a = \frac{M}{R_A \gamma_0 h_0} \Rightarrow F_a = 5,14 \text{ cm}^2 (\gamma_0 = 0,8);$$

$$M = \frac{q l^2}{8} = 8415 \text{ kgm}; \quad \xi = f \cdot (A_0);$$

$$A_0 = \frac{M}{R_b b h_0} = 0,0639.$$

According to the data in Table 1, it is established that to ensure the required operational reliability of the structure under consideration (Figure 1) (probability $P^* \geq 0,98$), it is necessary to adopt a reinforcement percentage μ^* of at least 0.25% (with $A_s^* \geq 6 \text{ cm}^2$).

If the reinforcement is selected using the deterministic calculation option [14], the required reinforcement percentage is $\mu_0 = 0.215\%$ ($A_s^0 \geq 5.15 \text{ cm}^2$).

The values (μ) and (μ_0) are close; however, the probabilistic method provides slightly higher reinforcement values, ensuring greater reliability ($P \geq 0.98$).

Figure 3 shows the dependence of reliability level on the reinforcement percentage μ for a reinforced concrete beam. The graph allows determining the required μ to ensure a given probability of failure-free operation (0.684–0.9855) within the range of 0.2–0.25%.

Conclusions

1. Using a reinforced concrete beam as an example, a methodology for assessing structural reliability at the design stage is proposed.

2. The probabilistic approach considers load-bearing capacity parameters as random variables and allows determining the probability of failure-free operation.

3. The results show that the probabilistic method provides a higher reliability level compared to the deterministic approach.

4. It is established that to ensure $P \geq 0.98$, the required reinforcement percentage is $\mu \geq 0.25\%$.

5. The proposed methodology can be applied in the design and assessment of building structures.

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Studying the reinforcement percentage in a reinforced concrete beam effect on its failure-free operation

μ (%)	A_s , cm^2	F_{S_r} , MHM	F_q , MHM	$q=F_S-F_q$	$\frac{\partial q}{\partial R_{S,n}}$	$\frac{\partial q}{\partial R_{b,n}}$	$\frac{\partial q}{\partial q_n}$	$\frac{\partial q}{\partial v_n}$	$\frac{\partial q}{\partial h_0}$	$\frac{\partial q}{\partial l}$	S_q^2	S_q	$\beta_1 = \frac{q_l}{S_q}$	$P=\Phi(\beta_1)$
0,2	4,8	0,0908	0,08415	0,00665	0,000249	0,000348	(const)	(const)	0,1752	(const)	0,00019366	0,01392	0,478	0,684
0,21	5,04	0,09436		0,01081	0,00026016	0,0003837			0,18396		0,00013274	0,01152	0,9384	0,8372
0,22	5,28	0,09908		0,01493	0,0002715	0,0004377			0,19272		0,00013767	0,01173	1,2784	0,8982
0,23	5,52	0,10316		0,01901	0,00028264	0,0004602			0,20148		0,00014278	0,01194	1,5913	0,945
0,24	5,76	0,10721		0,02306	0,00029372	0,0005011			0,21024		0,00014811	0,01217	1,895	0,9709
0,25	6,0	0,11122		0,02707	0,0003047	0,000544			0,219		0,00015364	0,01237	2,184	0,9855

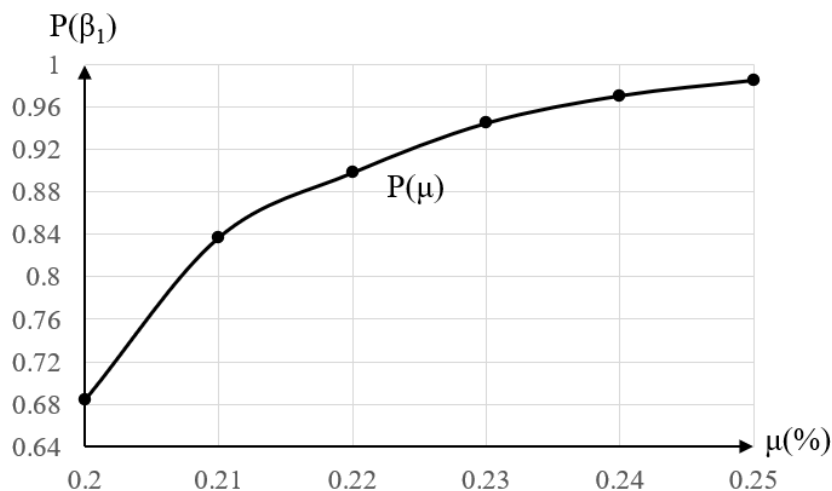


Figure 5 – FFO probability dependence on the reinforcement percent (μ) of the beam

REFERENCES

1. Armen Der Kiureghian. Structural and System Reliability. Cambridge University Press, 2022. 610 c. ISBN 1108834140, 9781108834148
2. Liu Xuan, Furrer David, Kusters Jared, Holmes Jack. Vision 2040: A Roadmap for Integrated, Multiscale Modeling and Simulation of Materials and Systems. 2018. P. 168.
3. Соловьева А.А., Смирнов В.А., Соловьев С.А. Моделирование неопределенности статистических данных р-блоками при вероятностном анализе надежности строительных конструкций покрытий // Сейсмостойкое строительство. Безопасность сооружений. 2024. № 4. С. 08-22. <https://doi.org/10.37153/2618-9283-2024-4-08-22>.
4. Taubaldy I., Akhmetov D., Rakisheva A. A comprehensive approach to concrete strength testing: combining destructive, non-destructive and wireless sensor methods // Bulletin of the L.N. Gumilev Eurasian National University. Series: Technical Sciences and Technologies. 2023. No. 1 (142). Pp. 98-106. <https://doi.org/10.32523/2616-7263-2023-142-1-98-106>.
5. Altigenov U., Bespaev A., Tulebekova A., Zharkenov Ye. Experimental evaluation of the strength of the floor slab of a volume block // Bulletin of the L.N. Gumilev Eurasian National University. Series: Technical Sciences and Technologies. 2024. No. 3 (148). Pp. 68-78. <https://doi.org/10.32523/2616-7263-2024-148-3-68-78/>
6. P.Ni.Z. Yuan, Q. Han, X. Du, and J. Fu, 'Reliability assessment of civil structures with incomplete probability distribution information', Probabilistic Eng. Mech., vol. 74, 2023, <https://doi.org/10.1016/j.probengmech.2023.103541>.
7. P. Zabojszcza, U. Radoń, and P. Tazowski, 'Robust and reliability-based design optimization of steel beams', Arch. Civ. Eng., vol. 69, no. 4, pp. 125-140, 2023, <https://doi.org/10.24425/ace.2023.147651>.
8. Райзер В.Д. (2010). Теория надежности сооружений. Москва: АСВ.
9. Hibbeler R.C. Structural Analysis. 10th ed. Pearson; 2017. 912p.
10. Melchers R.E., Beck A.T. Structural Reliability Analysis and Prediction. 3rd ed. Wiley; 2018. 514p.
11. Купняк О.Г. (2011). Железобетонные и каменные конструкции. Москва: АСВ.
12. EN 1990: Eurocode – Basis of Structural Design. Brussels: CEN; 2002 (updated versions).
13. Моисеенко Р.П. (2014). Начальная надежность железобетонной балки; Методические указания с выполнением расчетно-графической работой. Томский государственный архитектурный строительный университет.
14. Nilson A.H., Darwin D., Dolan C.W. Design of Concrete Structures. 15th ed. McGraw-Hill; 2016.

Темірбетон арқалықтарының көтеру сенімділігін ықтималдық негізінде зерттеу

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Аңдатпа. Темірбетон конструкцияларының сенімділігін ықтималдық әдістерімен бағалаудың теориялық және қолданбалы негіздері қарастырылады. Құрылыс конструкцияларының тұрғызу кезеңіндегі сенімділігін бағалау әдістемесі ұсынылған, ол көтергіштік қабілетті анықтайтын факторлық белгілердің статистикалық өңделуіне негізделген ықтималдық тәсілге сүйенеді. Сонымен қатар, ықтималдық әдістерді қолдану конструкциялардың сенімділік деңгейін арттыруға мүмкіндік беретіні анықталды. Алынған нәтижелерді құрылыс нысандарын жобалау кезеңінде құрылыс конструкцияларының сенімділігін бағалау үшін пайдалануға болады.

Кілт сөздер: темірбетон конструкциялары, иілуге жұмыс істейтін арқалық, сенімділік, ақаусыз жұмыс істеу ықтималдығы, сенімділік индексі, ықтималдық есептеу әдісі, статистикалық сызықтандыру, бетонды арматуралау, көтеру қабілеттілігі, жобалау.

Вероятностное исследование несущей надежности железобетонных балок

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Аннотация. В статье рассматриваются теоретические и прикладные основы вероятностной оценки надёжности железобетонных конструкций. Предложена методика оценки надёжности конструкции на стадии возведения с использованием вероятностного подхода, основанного на статистической обработке факторных признаков, определяющих несущую способность. Предложены расчётные зависимости, определяющие вероятности безотказной работы на основе индекса надёжности; при этом выполнено сравнение результатов детерминированного и вероятностного подходов. Установлено, при этом что применение вероятностных методов позволяет повысить уровень надёжности конструкций. Приведённые результаты могут быть использованы для оценки надёжности строительных конструкций на стадии проектирования строительных объектов.

Ключевые слова: железобетонные конструкции, изгибаемая балка, надёжность, вероятность безотказной работы, индекс надёжности, вероятностный метод расчёта, статистическая линеаризация, армирование, несущая способность, проектирование.

REFERENCES

1. Der Kiureghian A. Structural and system reliability. Cambridge: Cambridge University Press; 2022. 610 p. ISBN 9781108834148.
2. Liu X., Furrer D., Kusters J., Holmes J. Vision 2040: A roadmap for integrated, multiscale modeling and simulation of materials and systems; 2018. 168 p.
3. Solov'eva A.A., Smirnov V.A., Solov'ev S.A. Modelirovanie neopredelennosti statisticheskikh dannykh r-blokami pri veroyatnostnom analize nadezhnosti stroitel'nykh konstruktsiy pokrytiy (Modeling uncertainty of statistical data using r-blocks in probabilistic reliability analysis of structural coverings). Seysmostoykoe stroitel'stvo. Bezopasnost' sooruzheniy. 2024; 4: 8-22. <https://doi.org/10.37153/2618-9283-2024-4-08-22>.
4. Taubaldy I., Akhmetov D., Rakisheva A. A comprehensive approach to concrete strength testing: combining destructive, non-destructive and wireless sensor methods. Bulletin of the L.N. Gumilev Eurasian National University. Technical Sciences and Technologies. 2023; 1 (142): 98-106. <https://doi.org/10.32523/2616-7263-2023-142-1-98-106>.
5. Altigenov U., Bespaev A., Tulebekova A., Zharkenov Ye. Experimental evaluation of the strength of the floor slab of a volume block. Bulletin of the L.N. Gumilev Eurasian National University. Technical Sciences and Technologies. 2024; 3 (148): 68-78. <https://doi.org/10.32523/2616-7263-2024-148-3-68-78>.
6. Yuan P.N.Z., Han Q., Du X., Fu J. Reliability assessment of civil structures with incomplete probability distribution information. Probabilistic Engineering Mechanics. 2023; 74: 103541. <https://doi.org/10.1016/j.probengmech.2023.103541>.
7. Zabojszcza P., Radoń U., Tuzowski P. Robust and reliability-based design optimization of steel beams. Archives of Civil Engineering. 2023; 69 (4): 125-140. <https://doi.org/10.24425/ace.2023.147651>.
8. Raizer V.D. Teoriya nadezhnosti sooruzheniy (Theory of structural reliability). Moscow: ASV; 2010. ISBN 978-5-93093-746-7.
9. Hibbeler R.C. Structural Analysis. 10thed. Pearson; 2017. 912 p.
10. Melchers R.E., Beck A.T. Structural Reliability Analysis and Prediction. 3rded. Wiley; 2018. 514 p.
11. Kupnyak O.G. Zhelezobetonnye i kamennye konstruksii (Reinforced concrete and masonry structures). Moscow: ASV; 2011.
12. EN 1990: Eurocode – Basis of Structural Design. Brussels: CEN; 2002 (updated versions).
13. Moiseenko R.P. Nachal'naya nadezhnost' zhelezobetonnoy balki. Metodicheskie ukazaniya s vypolneniem raschetno-graficheskoy raboty (Initial reliability of reinforced concrete beam. Guidelines with calculation and graphical work). Tomsk: Tomsk State University of Architecture and Building; 2014.
14. Nilson A.H., Darwin D., Dolan C.W. Design of Concrete Structures. 15thed. McGraw-Hill; 2016.