

Modeling Temperature Distribution in Heating Network Considering Heat Exchange with Environment

BEKIBAYEV Timur, Master's Degree, Head of Department, timur_bekibaev@mail.ru,
RAMAZANOVA Gaukhar, Cand. of Phys. and Math. Sc., Deputy Head of the Scientific and Production Laboratory, gaukhar.ri@gmail.com,
***KUDAIBERGEN Azamat**, Master's Degree, Researcher, kudaibergen.azamatd28@gmail.com, NPJSC «Kazakh National Research Technical University named after K.I. Satpayev», 22a Satpayev Street, Almaty, Kazakhstan,
 *corresponding author.

Abstract. The results of modeling the temperature distribution in the main pipelines of the heating network taking into account heat exchange with the environment, are presented. The temperature distribution along the pipe section length (supply or return) between the nodes of the heating network is described by the Shukhov formula. The values of the ambient temperature and the thermal resistance from the outer surface of the pipe to the external environment, according to the Shukhov formula, were determined based on the type of pipeline installation (above-ground, underground non-channel, and underground channel). With above-ground laying, the influence of the temperature field of the parallel pipe (adjacent) on the pipe in question was not taken into account. With underground parallel pipe laying, the temperature at the end of the section requires the calculated value of the water temperature of the adjacent pipe, which also depends on the temperature of the pipe in question. Therefore, the calculation is an iterative process of calculating the final temperature of the section until the required accuracy is achieved.

Keywords: modeling, temperature distribution, heating network, heat exchange, thermal resistance, pipeline, above-ground pipeline installation, underground pipeline installation, ambient temperature, Shukhov formula.

Introduction

As stated in the International Energy Agency (IEA) 2023 report, half of the world's energy demand was caused by space and water heating [1]. At the same time, the combustion of fossil fuels meets 60% of the energy for heating buildings [1]. District heating systems are widely used to meet the needs of heating, hot water and cooling through a network of insulated pipes [2]. The expansion and modernization of district heating systems in urban areas is a concrete way to significantly reduce fossil fuel consumption and accelerate decarbonization in the heating sector [3].

Underground pipelines of thermal networks, arranged in parallel or twin-pipe configurations, are used in district heating systems. Heat losses from heat distribution through pipelines are one of the biggest difficulties when distributing the temperature of the heating network. Optimizing the thickness of the insulation layer relative to the pipe diameter is

an effective solution to this problem. The most suitable insulation thickness should correspond to the minimum heat losses at relatively low investment costs. Analytical calculations and numerical simulations have been used to determine heat losses in numerous studies. However, studies that allow measuring operating parameters (flow rate, temperature, fluid pressure in the supply and return pipelines) are clearly insufficient.

The article [4] presents an analysis of heat losses from pre-insulated pipes and two-pipe systems in the heat supply network. It also compares the calculations of heat losses in the soil obtained using an analytical solution (one-dimensional model) with experiments on a special experimental setup. Calculations of heat losses of two-pipe systems and their analogues in one parallel isolated system were performed for several options [4]. The pipe insulation was made of polyurethane foam with a thickness of 30.85 mm. Thermal conductivity

of the insulating material was found during the study and used in the calculations of temperature distribution [4]. The model calculations were compared with the measurement results in a laboratory setup and showed compliance with an accuracy of up to 8% depending on the type of heating pipe. The developed mathematical model was used to determine heat losses in the heating network [4]. It is shown that the payback period for investments in insulation for twin-pipe and single-pipe district heating systems is estimated to be within 5 years.

The distortion of the temperature profile throughout the district heating network was studied through the dynamic performance of district heating systems [5]. The nodal modeling method was applied for the analysis [5]. Commercial TERMIS software was also used in this study. The calculations revealed sudden temperature changes in the supply and transient conditions for the appearance of the thermal wave in the district heating system. The simulation results were compared with consumer data in district heating systems [5]. The analysis of calculations showed uneven propagation of the thermal wave at different locations in the heating network [5].

Heat distribution through interconnected and insulated pipes, linking heat sources with local consumers, provides an efficient solution for district heating [6]. Integrating the heating network with renewable heat sources and heat storage systems further enhances the attractiveness of district heating. Complex dynamic models are required for this purpose, which are computationally expensive, especially in large-scale networks. Artificial neural networks are proposed to reduce the computation time associated with dynamic simulations of district heating systems [6].

A district heating model with multiple heat sources to study the impact of an additional integrated heat source and the effects of lowering the supply temperature on the performance of the network is presented in [7]. It is shown that lowering the supply temperature can improve the overall system efficiency and should also be considered for the sustainable transformation of existing district heating systems [7].

The intelligent district heating system has great development potential and broad market prospects and shows energy saving, regulation and control of the heating network [8]. The concept of the intelligent district heating system is discussed in detail, as well as the corresponding function of each component [8]. Technologies that can be well integrated into this intelligent system are presented and evaluated in [8].

The work [9] provides an overview of the modeling and optimization of district heating systems, taking into account heat distribution within the network. The modeling of the main district heating components is discussed, including heat sources, end users, and the distribution network [9]. Key optimization criteria for the design and operation of the distribution network are formulated [9].

The main concepts and principles of modeling district heating networks are presented in [10]. An overview of mathematical models for district heating networks is provided, with a focus on potential research areas and the development of tools [10].

The work [11] proposes a dynamic thermal simulation model for a thermal network with central heating equipment, which can evaluate node temperatures and calculate heat losses in pipes. To accelerate calculations, variable time steps were used through lazy evaluation and prioritization, as well as a method for eliminating redundant sampling points [11]. The model was tested and demonstrated consistency with actual measurements [11].

As seen from the above-mentioned works, determining heat losses in pipelines is crucial for establishing the temperature regime of the heating network.

This work presents the results of modeling the temperature distribution in the heating network, taking into account heat exchange with the environment.

Methodology

Calculation of water temperature in pipelines depends on heat exchange of the coolant with the environment. Below we consider the method of determining the temperature in the supply pipeline under the assumption that movement and heat exchange occur in a stationary mode.

The temperature distribution along the pipe section length (supply or return) between the heating network nodes is described by the following heat transfer equation:

$$u \frac{dT}{dx} + \frac{4K}{\rho D C_p} (T - T_{env}) = 0, \quad (1)$$

where x is the coordinate along the pipeline section length; T is the coolant temperature (water); ρ is the water density; C_p is the water specific heat capacity; D is the pipeline internal diameter; u is the average flow velocity across the pipe cross section, which is expressed in terms of water flow rate q : $u = \frac{q}{0.25\pi D^2}$; K is the heat transfer coefficient from the flow of coolant to the environment; T_{env} is the ambient temperature.

This equation has the following solution, known as the Shukhov formula:

$$T(x) = T_{env} + (T_0 - T_{env})e^{-\frac{\pi \rho K D x}{q C_P}}. \quad (2)$$

The value of K is determined from the following relationship:

$$\frac{1}{K\pi D} = R = R_{in} + R_{ins} + R_{out}, \quad (3)$$

where R is the total thermal resistance of the heat transfer pipe;

R_{in} is the thermal resistance of heat transfer from the coolant to the pipe wall;

R_{ins} is the thermal resistance of the pipe insulation coating;

R_{out} is the thermal resistance from the pipe outer surface to the external environment.

The value of R_{in} was determined using the Nusselt number Nu , which is expressed in terms of the Reynolds number Re and the Prandtl number Pr :

$$R_{in} = \frac{1}{\lambda_w \pi Nu}, \quad Nu = 0,021 (Re)^{0,8} (Pr)^{0,43}, \quad (4)$$

$$Re = \frac{uD}{\nu}, \quad Pr = \frac{\nu \rho C_P}{\lambda_w},$$

where λ_w is the water thermal conductivity coefficient; ν is the water kinematic viscosity.

The value of R_{ins} was determined using the insulation coating parameters:

$$R_{ins} = \frac{1}{2\pi \lambda_{ins}} \ln \frac{D_2}{D_1}, \quad (5)$$

where D_1, D_2 are the inner and outer diameters of the insulating coating layer of the pipeline; λ_{ins} is the thermal conductivity coefficient of the insulation layer.

The value of T_{env} in formula (2) and the value of R_{out} in relation (3) were determined depending on the type of pipeline installation (aboveground, underground without a channel, and underground with a channel). As a rule, each section of the heating network contains two pipes: supply and return pipes, in which there are different temperatures of the coolant. Unlike underground installation, above-ground installation did not take into account the influence of the temperature field of a parallel laid pipe (adjacent) on the pipe in question.

For above-ground pipeline installation, the outside air was considered as the environment, the value of R_{out} was calculated using the following formula:

$$T_{env} = T_{air}, \quad R_{out} = \frac{1}{\alpha_{air} \pi D_2}, \quad \alpha_{air} = 11,6 + \sqrt{w}, \quad (6)$$

where α_{air} is the heat transfer coefficient from the pipe outer surface to the surrounding air; w is the wind speed; T_{air} is the outside air temperature.

For underground ductless pipeline laying,

the underground soil was considered as the environment, the value of R_{out} was calculated using the following relationship, which takes into account the mutual influence of the current and adjacent pipes:

$$T_{env} = T_{gr}, \quad \frac{T_{avg} - T_{gr}}{R_{out}} =$$

$$= \frac{(T_{avg} - T_{gr}) \bar{R}_0 - (\bar{T}_{avg} - T_{gr}) R_0}{R_0 \bar{R}_0 - R_{inf}^2}, \quad (7)$$

$$R_{inf} = \frac{1}{2\pi \lambda_{gr}} \ln \sqrt{1 + \left(\frac{2H}{s}\right)^2},$$

where T_{gr} is the soil temperature; T_{avg}, \bar{T}_{avg} are the average temperatures of the considered and adjacent pipes, respectively; R_{inf} is the additional thermal resistance of heat transfer between two pipes in the soil; λ_{gr} is the thermal conductivity coefficient of the surrounding soil; H is the pipeline depth to its axis; s is the distance between the axes of the considered and adjacent pipes; R_0, \bar{R}_0 are the total thermal resistances of the considered and adjacent pipes without taking into account the mutual influence of their temperature fields.

The values of R_0, \bar{R}_0 are calculated using formula (3), where the corresponding values of R_{out}, \bar{R}_{out} are equal to the soil thermal resistance R_{gr} , which is determined using the Forchheimer formula:

$$R_{gr} = \frac{1}{2\pi \lambda_{gr}} \ln \left(\frac{2H}{D_2} + \sqrt{\left(\frac{2H}{D_2}\right)^2 - 1} \right).$$

For underground channeled pipeline installation, the air within the channel was considered as the environment, and the value of R_{out} was calculated as the sum of the thermal resistances of consecutive heat transfers within the channel:

$$T_{env} = T_{air}^{can}, \quad R_{out} = R_{air}^{can} + R_{wall}^{can} + R_{gr}^{can},$$

$$R_{air}^{can} = \frac{1}{\alpha_{air}^{can} \pi D_2}, \quad R_{wall}^{can} = \frac{1}{\alpha_{air}^{can} \pi D_{can}}, \quad (8)$$

$$D_{can} = \frac{2d_{can}h_{can}}{d_{can} + h_{can}}, \quad R_{gr}^{can} = \frac{\ln \left(3,5 \frac{Hh_{can}}{d_{can}^2} \right)}{\lambda_{gr} \left(5,7 + \frac{d_{can}}{2h_{can}} \right)},$$

where $R_{air}^{can}, R_{wall}^{can}, R_{gr}^{can}$ are the values of the corresponding thermal resistances: heat transfer from the surface of the insulated pipeline into the air space of the channel, heat transfer from the air in the channel to the soil, heat transfer of the soil around the channel; α_{air}^{can} is the air heat transfer coefficient in the channel, according to [13] the value of $8 \text{ W}/(\text{m}^2 \cdot ^\circ\text{C})$ was taken; $D_{can}, d_{can}, h_{can}$ are the equivalent diameter, width and height of the channel;

T_{air}^{can} is the channel air temperature, which was calculated based on the following relationship for thermal resistances:

$$\frac{T_{air}^{can} - T_{avg}}{R_{ins} + R_{air}^{can}} + \frac{T_{air}^{can} - \bar{T}_{avg}}{\bar{R}_{ins} + \bar{R}_{air}^{can}} + \frac{T_{air}^{can} - T_{gr}}{R_{wall}^{can} + R_{gr}^{can}} = 0, (9)$$

where $\bar{R}_{ins}, \bar{R}_{air}^{can}$ are the values of the corresponding thermal resistances (insulation coating and heat transfer from the pipe surface into the air space of the channel), which are calculated using formulas (6), (8) relative to the adjacent pipe parameters.

As can be seen from formulas (7)-(9), the calculation of the temperature at the end of an underground pipe section requires the calculated value of the water temperature in the adjacent pipe, which also depends on the temperature in the pipe in question. Therefore, when laying the underground pipe, an iterative process is performed to calculate the final temperature of the section until the required accuracy is achieved.

Thus, using the initial temperature T_0 at the pipe inlet of the heating network section and the flow rate of the coolant q through this section, the corresponding final temperatures T_{end} were determined using formulas (2)-(9).

The temperature distribution across the entire heating network was determined using a breadth-first search (graph traversal method), starting from the heating network nodes (CHP, boiler houses) for which the outlet temperatures were specified. The traversal of the heating network nodes was performed in the positive direction of flow ($q > 0$). When traversing the heating network nodes, the temperature T_0 at the inlet of subsequent sections of the heat-

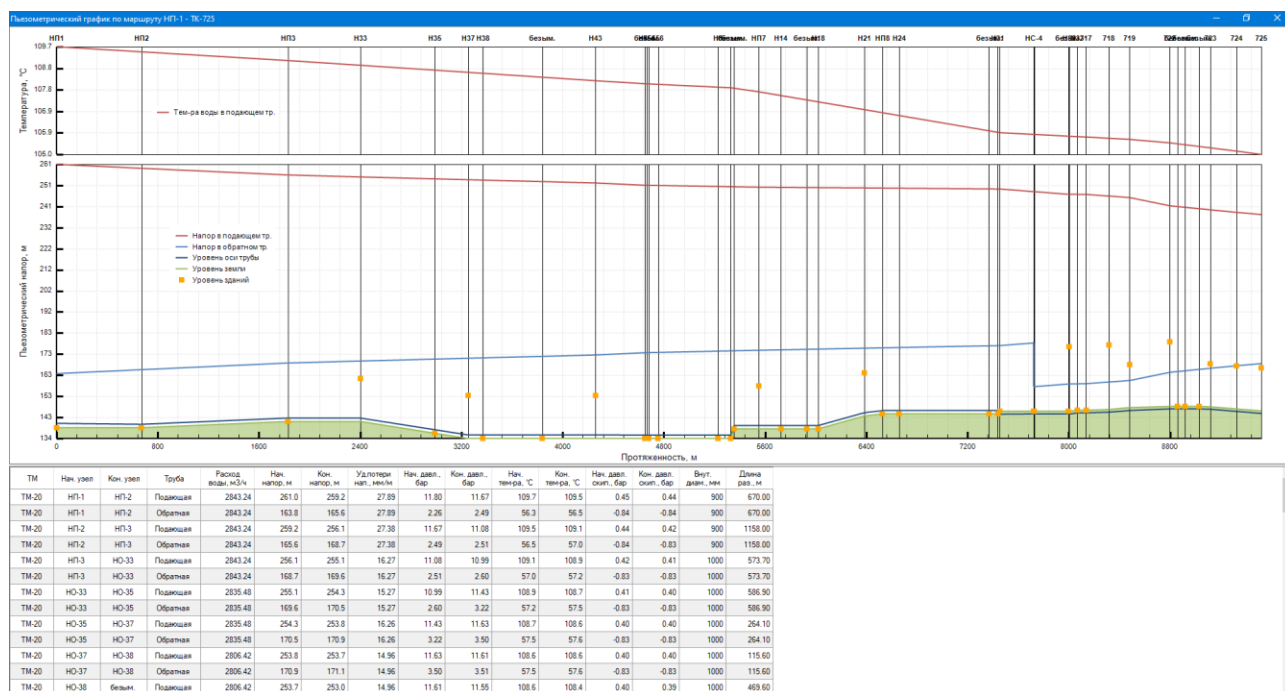
ing network (the temperature at the outlet of the initial node of the section) was calculated based on the heat energy balance:

$$T_0 = \frac{\sum_{i=1}^n (q_i (T_{end})_i)}{\sum_{i=1}^n q_i} - \Delta T, (10)$$

where n is the number of h^2 eating network nodes from which the flow comes to the current node; $q_i, (T_{end})_i$ is the values of water flow rate and final temperature at the i -th section of the pipe entering the current node; ΔT is the temperature loss at the current node. If the current node is a consumer in the heat supply system, then $\Delta T > 0$, otherwise $\Delta T = 0$.

Figure shows the calculated data for the temperature and piezometric head distribution in the supply and return pipelines of the heating network. Heat exchange with the environment leads to a decrease in the water temperature in the supply pipeline from 109.7°C to 105°C along the length $L = 9520$ m (see figure). It can be seen that along the length of the pipeline to point $L_1 = 5280$ m, the distribution of the hot water temperature is almost linear, starting from this point, the temperature decreases to point $L_2 = 7540$ m. Then the drop in the hot water temperature stabilizes (see figure).

The same figure shows the piezometric heads in the supply (red line) and return (blue line) pipelines. In the supply pipeline, the pressure decreases due to the withdrawal of hot water for heat supply to consumers, and in the



Temperature distribution in the supply and return main pipelines of the heating network

return pipeline, on the contrary, the pressure is restored due to the inflow of cooled water (see figure). The sharp decrease in pressure at point $L_3 = 7700$ m of the return pipeline is associated with local resistance at this point.

The program code for solving the problem of temperature distribution in the heating network based on the process of heat exchange with the environment was written in C# and implemented as a separate software module of the System.

Conclusion

Determining heat losses from heat distribution through pipelines is one of the biggest difficulties in distributing the temperature of the heating network. The paper presents the results of heat loss calculations depending on the pipe installation method in the heating network. The temperature distribution across the entire heating network was determined using a breadth-first search (graph traversal meth-

od), starting from the heating network nodes (CHP, boiler houses) for which the outlet temperatures were specified. The traversal of the heating network nodes was carried out in the positive flow direction ($q > 0$). When traversing the heating network nodes, the temperature value T_0 at the inlet of subsequent sections of the heating network (the temperature at the outlet of the initial node of the section) was calculated based on the heat energy balance.

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Declaration of competing interest

Authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Қоршаған ортамен жылу алмасуды ескере отырып, жылу желісіндегі температураның таралуын модельдеу

БЕКИБАЕВ Тимур Талғатович, магистр, бөлім меңгерушісі, timur_bekibaev@mail.ru,
РАМАЗАНОВА Гаухар Избасаровна, ф.-м.ғ.к., ғылыми-өндірістік зертхана
меңгерушісінің орынбасары, gaukhar.ri@gmail.com,
***ҚҰДАЙБЕРГЕН Азамат Доскелдіұлы**, магистр, ғылыми қызметкер,
kudaibergen.azamatd28@gmail.com,
«Қ.И. Сәтбаев атындағы Қазақ ұлттық техникалық зерттеу университеті» КеАҚ, Сәтбаев
көшесі, 22а, Алматы, Қазақстан,
*автор-корреспондент.

Аңдатпа. Мақалада қоршаған ортамен жылу алмасуды ескере отырып, жылу желісінің магистральдық құбырларындағы температураның таралуын модельдеу нәтижелері келтірілген. Жылу желісінің түйіндері арасындағы құбыр учаскесінің ұзындығы бойынша (беру немесе қайтару) температураның таралуы Шухов формуласымен сипатталады. Қоршаған орта температурасының мәндері және құбырдың сыртқы бетінен сыртқы ортаға жылу кедергісі Шухов формуласы бойынша құбырды орнату түріне (жер үстінде, жер асты арнасыз және жер асты каналында) байланысты анықталды. Жер үсті төсеу кезінде параллель құбырдың (іргелес) температура өрісінің қарастырылып отырған құбырға әсері ескерілмеді. Жер асты параллель құбырды төсеу кезінде секцияның соңындағы температура іргелес құбырдың су температурасының есептік мәнін талап етеді, бұл да қарастырылып отырған құбырдың температурасына байланысты. Демек, есептеу қажетті дәлдікке жеткенше қиманың соңғы температурасын есептеудің қайталанатын процесі болып табылады.

Кілт сөздер: модельдеу, температураның таралуы, жылу желісі, жылу алмасу, жылу кедергісі, құбыр желісі, жер үстіндегі құбырды орнату, жер асты құбырын орнату, қоршаған орта температурасы, Шухов формуласы.

Моделирование распределения температуры в тепловой сети с учетом теплообмена с окружающей средой

БЕКИБАЕВ Тимур Талғатович, магистр, зав. отделом, timur_bekibaev@mail.ru,
РАМАЗАНОВА Гаухар Избасаровна, к.ф.-м.н., зам. зав. научно-производственной лабораторией, gaukhar.ri@gmail.com,
***ҚҰДАЙБЕРГЕН Азамат Доскелдыұлы**, магистр, научный сотрудник,
kudaibergen.azamatd28@gmail.com,
НАО «Казахский национальный исследовательский технический университет имени К.И. Сатпаева», ул. Сатпаева, 22а, Алматы, Казахстан,
*автор-корреспондент.

Аннотация. Представлены результаты моделирования распределения температуры в магистральных трубопроводах тепловой сети с учетом теплообмена с окружающей средой. Распределение температуры по длине участка трубы (подача или обратка) между узлами тепловой сети описывается формулой Шухова. Значения температуры окружающей среды и термического сопротивления от наружной поверхности трубы к внешней среде по формуле Шухова определялись исходя из типа прокладки трубопровода (надземная, подземная бесканальная и подземная канальная). При надземной прокладке влияние температурного поля параллельной трубы (смежной) на рассматриваемую трубу не учитывалось. При подземной параллельной прокладке труб для определения температуры в

конце участка необходимо расчетное значение температуры воды смежной трубы, которая также зависит от температуры рассматриваемой трубы. Поэтому расчет представляет собой итерационный процесс вычисления конечной температуры участка до достижения требуемой точности.

Ключевые слова: моделирование, распределение температуры, тепловая сеть, теплообмен, термическое сопротивление, трубопровод, надземная прокладка трубопровода, подземная прокладка трубопровода, температура окружающей среды, формула Шухова.

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