

# Review and Synthesis of the Main Contour of the Adaptive Control System for Unstable and Deterministic Chaotic Processes in the "Swallow Tail" Class of Catastrophes

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**Abstract.** The research of the last century revealed a wide variety of dynamics of nonlinear systems and led to one of the most important discoveries of the twentieth century in nonlinear dynamical systems – deterministic chaos and a strange attractor. An urgent problem in the conditions of uncertainty of the parameters of the control object and external influences, for the management of unstable and deterministic chaotic processes is the use of adaptation methods. At the same time, the reference model with the desired dynamics and the main control loop of the adaptive system are constructed in the class of «Dovetail» catastrophes, and the aperiodic robust stability of the reference model with the desired dynamics and the main control loop of the adaptive system with an increased potential for robust stability is investigated by the gradient-velocity method of the Lyapunov vector functions. From the conditions of aperiodic robust stability of a generalized tunable object, the tunable coefficients are calculated and the control goal is achieved. The solution of the problem of synthesis of the main contour of the adaptive control system of unstable and deterministic chaotic processes with  $m$  – inputs and  $n$  – outputs in the class of «Dovetail» catastrophes is proposed.

**Keywords:** adaptive, deterministic chaos, dovetail, contour synthesis, robust stability, invariant, gradient system, scalar product, catastrophe theory, static accuracy, system of inequalities, dynamic systems.

## Introduction

Currently, it is generally recognized that real control objects are nonlinear and deterministic chaos with the generation of a «strange attractor» is an internal property of any nonlinear deterministic dynamical system [1]. In linearized dynamical systems, this manifests itself as a loss of robust stability.

Synthesis of the main contour of the adaptive control system for unstable and deterministic chaotic processes with known parameters is inextricably linked with ensuring stability, robustness and quality of control. In adaptive control systems, external influences are compensated, i.e. the control system becomes invariant with respect to external influences [2], and the choice of a reference model with the desired dynamics and synthesis of the main control loop in the class of «swallow tail» disasters provides robust stability, and the desired dynamics of the reference model, the study and synthesis of all adaptive

control subsystems, by the gradient-velocity method of the Lyapunov vector functions [3] guarantees aperiodic robust stability of systems of a given control quality.

The problem of synthesis of the main control loop with known values of the parameters of the control object is solved by the gradient-velocity method of the vector of Lyapunov functions [4].

Methods based on the application of Lyapunov functions are the main ones in the research and synthesis of adaptive control systems. But at present, these methods serve only as a tool for theoretical research and cannot provide answers to the basic questions of research and synthesis of adaptive regulators in real conditions. The main obstacle in this case is the lack of a universal approach to the construction of Lyapunov functions.

In this paper, a dynamic system is represented as gradient systems, and Lyapunov functions as poten-

tial functions [5], this allows us to write:

$$\frac{dx^i}{dt} = -\frac{\partial V}{\partial x^i}, \quad i = 1, \dots, n.$$

The application of these relations and Morse's lemma from the theory of catastrophes makes it possible to construct a set of Lyapunov functions and solve synthesis problems with known values of the parameters of the control object.

Feedback gain coefficients are calculated from the conditions of aperiodic robust stability of the reference model with the desired dynamics and the main contour of the adaptive control system for unstable processes.

**Methods and results of the study**

1. The problem of synthesis of the main contour of an adaptive control system for unstable and deterministic chaotic processes in the class of «dovetail» catastrophe functions for a system with m-inputs and n-outputs is considered. The problem is solved under the assumption that the parameters of the control object are known and equal to the same parameters of the reference model.

Let the control system of m inputs and n outputs be described by the equation of state in standard form:

$$\frac{dx}{dt} = Ax + Bu. \tag{1}$$

where  $x(t) \in R^n$  vector the status of the control system,  $u(t) \in R^m$  vector functions of management, and  $A \in R^{n \times n}$  matrix of the control object of dimension  $n \times n$ ,  $B \in R^{m \times n}$  the control matrix of dimension  $m \times n$ .

Let the matrices  $A$  and  $B$  have the following form:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & 0 & 0 & \dots & 0 \\ 0 & b_{22} & 0 & \dots & 0 \\ 0 & 0 & b_{33} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & b_{nn} \end{pmatrix}.$$

The control law is given in the form of three-parameter structurally stable maps (dovetail catastrophes) [9, 10, 11]:

$$u_i = \frac{1}{5}x_i^5 - \frac{1}{3}k_i^1 x_i^3 - \frac{1}{2}k_i^2 x_i^2 + k_i^3 x_i, \quad i = 1, \dots, n. \tag{2}$$

The system (1), taking into account the control law (2), is written in an expanded form.

$$\begin{cases} \frac{dx_1}{dt} = -\frac{1}{5}b_{11}x_1^5 - \frac{1}{3}b_{11}k_1^1 x_1^3 - \frac{1}{2}b_{11}k_1^2 x_1^2 + \\ + (b_{11}k_1^3 + a_{11}) \cdot x_1 + a_{12}x_2 + \dots + a_{1n}x_n, \\ \frac{dx_2}{dt} = a_{21}x_1 - \frac{1}{5}b_{22}x_2^5 - \frac{1}{3}b_{22}k_2^1 x_2^3 - \frac{1}{2}b_{22}k_2^2 x_2^2 + \\ + (b_{22}k_2^3 + a_{22})x_2 + \dots + a_{2n}x_n, \\ \dots \\ \frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + \dots - \frac{1}{5}b_{nn}x_n^5 - \\ - \frac{1}{3}b_{nn}k_n^1 x_n^3 - \frac{1}{2}b_{nn}k_n^2 x_n^2 + (b_{nn}k_n^3 + a_{nn})x_n. \end{cases} \tag{3}$$

The steady states of the system (3) are determined by [10, 11, 12]:

$$x_{1s}^1 = 0, x_{2s}^1 = 0, \dots, x_{ns}^1 = 0, \tag{4}$$

$$x_{is}^2 = \sqrt[4]{\frac{k_i^3 + a_{ii}}{4}}, k_i^2 = 6\left(\sqrt[4]{\frac{1}{4}(k_i^3 + a_{ii})}\right)^3, \tag{5}$$

$$k_i^1 = \sqrt{\frac{1}{4}(k_i^3 + a_{ii})}, i = 1, \dots, n.$$

2. Study of the stability of the stationary state (4).

From the equation of state, we determine the components of the gradient vector of the Lyapunov vector function  $V(x) = (V_1(x), V_2(x), \dots, V_n(x))$  [6, 12, 16]:

$$\begin{cases} \frac{\partial V_i(x)}{\partial x_{i+1}} = -a_{i1}x_1, \frac{\partial V_i(x)}{\partial x_2} = a_{i2}x_2, \dots, \frac{\partial V_i(x)}{\partial x_i} = \\ = \frac{1}{5}b_{ii}x_i^5 + \frac{1}{3}b_{ii}k_i^1 x_i^3 + \frac{1}{2}b_{ii}k_i^2 x_i^2 - (b_{ii}k_i^3 + a_{ii})x_i, \\ \frac{\partial V_i(x)}{\partial x_{i+1}} = -a_{i,i+1}x_{i+1}, \dots, \frac{\partial V_i(x)}{\partial x_n} = a_{in}x_n, i = 1, \dots, n. \end{cases} \tag{6}$$

From (3) we determine the components of the decomposition of the velocity vector by coordinates [6]:

$$\begin{cases} \left(\frac{dx_i}{dt}\right)_{x_i} = a_{i1}x_1, \left(\frac{dx_i}{dt}\right)_{x_2} = a_{i2}x_2, \dots, \left(\frac{dx_i}{dt}\right)_{x_i} = -\frac{1}{5} \cdot \\ \cdot b_{ii}x_i^5 - \frac{1}{3}b_{ii}k_i^1 x_i^3 - \frac{1}{2}b_{ii}k_i^2 x_i^2 + (b_{ii}k_i^3 + a_{ii}) \cdot x_i, \\ \left(\frac{dx_i}{dt}\right)_{x_{i+1}} = a_{i,i+1}x_{i+1}, \left(\frac{dx_{i2}}{dt}\right)_{x_n} = a_{in}x_n, i = 1, \dots, n. \end{cases} \tag{7}$$

We calculate the full time derivative of the Lyapunov vector function as the scalar product of the gradient vector (6) by the velocity vector (7) [6, 12]:

$$\begin{aligned} \frac{dV(x)}{dt} &= \sum_{i=1}^n \left( \sum_{j=1}^n \frac{\partial V_i(x)}{\partial x_j} \left(\frac{dx_j}{dt}\right)_{x_j} \right) = \\ &= \sum_{i=1}^n - (a_{i1}x_1)^2 - (a_{i2}x_2)^2 - \dots - \left(\frac{1}{5}b_{ii}x_i^5 + \right. \\ &\left. + \frac{1}{3}b_{ii}k_i^1 x_i^3 + \frac{1}{2}b_{ii}k_i^2 x_i^2 - (b_{ii}k_i^3 + a_{ii}) \cdot x_i \right). \end{aligned} \tag{8}$$

It is obvious from (8) that the total time derivative of the Lyapunov vector function is guaranteed to be a sign-negative function, i.e. a sufficient condition for asymptotic Lyapunov stability for system (3) is guaranteed to be fulfilled. Using the gradient of the desired Lyapunov vector function (6), we construct the vector of the Lyapunov function in scalar form:

$$\begin{aligned} V(x) &= \frac{1}{30}b_{11}x_1^6 + \frac{1}{12}b_{11}k_1^1 x_1^4 + \frac{1}{6}b_{11}k_1^2 x_1^4 - \frac{1}{2}(b_{11} \cdot \\ \cdot k_1^3 + a_{11}) \cdot x_1^2 - \frac{1}{2}a_{12}x_2^2 - \frac{1}{2}a_{13}x_3 - \dots - \frac{1}{2}a_{1n}x_n^2 + \\ + \frac{1}{30}b_{22}x_2^6 + \frac{1}{12}b_{22}k_2^1 x_2^4 + \frac{1}{6}b_{22}k_2^2 x_2^4 - \frac{1}{2}(b_{22}k_2^3 + \\ + a_{22})x_2^2 - \frac{1}{2}a_{21}x_1^2 - \frac{1}{2}a_{23}x_3 - \dots - \frac{1}{2}a_{2n}x_n^2 - \dots - \\ - \frac{1}{2}a_{n1}x_1^2 - \frac{1}{2}a_{n2}x_2^2 - \frac{1}{2}a_{n3}x_3^2 - \dots + \frac{1}{30}b_{nn}x_n^6 + \\ + \frac{1}{12}b_{nn}k_n^1 x_n^4 + \frac{1}{6}b_{nn}k_n^2 x_n^4 - \frac{1}{2}(b_{nn}k_n^3 + a_{nn}) \cdot x_n^2. \end{aligned} \tag{9}$$

The conditions of positive or negative definiteness of function (9) are not obvious, so we will use Morse's lemma from the theory of catastrophes.

And we can represent it in the form of a quadratic form:

$$V(x) = -[(b_{11}k_1^3 + a_{11}) + a_{21} + a_{31} + \dots + a_{n1}]x_1^2 - [a_{12} + (b_{22}k_2^3 + a_{22}) + a_{32} + \dots + a_{n2}]x_2^2 - \dots - [a_{n1} + a_{n2} + a_{n3} + \dots + (b_{nn}k_n^3 + a_{nn})]x_n^2, \quad (10)$$

$$\begin{cases} -[(b_{11}k_1^3 + a_{11}) + a_{21} + a_{31} + \dots + a_{n1}] > 0 \\ -[a_{12} + (b_{22}k_2^3 + a_{22}) + a_{32} + \dots + a_{n2}] > 0 \\ \dots\dots\dots \\ -[a_{n1} + a_{n2} + a_{n3} + \dots + (b_{nn}k_n^3 + a_{nn})] > 0. \end{cases} \quad (11)$$

It should be noted that when the system of inequalities (11) is fulfilled, the stationary state (4) will exist and is stable. Under the conditions of violation of these inequalities (11), a new stationary state (5) appears. The stationary state (4) and (5) do not exist at the same time. It is proved by the results of the theory of catastrophes that the stationary state (5) exists and appears with the loss of stability of the stationary state (4). Such a remarkable property of structurally stable maps from the theory of catastrophes is used to expand the field of robust stability of the control system under conditions of parametric uncertainty [7].

3. We investigate the stability of the stationary state (5) by the gradient-velocity method of the vector of the Lyapunov function. For the study, the system (3) is represented in deviations relative to the stationary state (6):

$$\begin{cases} \frac{dx_1}{dt} = -\frac{1}{5}b_{11}x_1^5 - b_{11}\sqrt{\frac{b_{11}k_1^3 + a_{11}}{4}}x_1^4 - 2b_{11}\sqrt{\frac{b_{11}k_1^3 + a_{11}}{4}}x_1^3 - b_{11}\left(k_1^3 + \frac{a_{11}}{b_{11}}\right)x_1 - 6b_{11}\left(\sqrt{\frac{b_{11}k_1^3 + a_{11}}{4}}\right)^3x_1^2 + a_{12}x_2 + \dots + a_{1n}x_n, \\ \frac{dx_2}{dt} = -\frac{1}{5}b_{22}x_2^5 - b_{22}\sqrt{\frac{b_{22}k_2^3 + a_{22}}{4}}x_2^4 - 2b_{22}\sqrt{\frac{b_{22}k_2^3 + a_{22}}{4}}x_2^3 - 6b_{22}\left(\sqrt{\frac{b_{22}k_2^3 + a_{22}}{4}}\right)^3x_2^2 \times \\ \times x_2^2 - b_{22}\left(k_2^3 + \frac{a_{22}}{b_{22}}\right)x_2 + a_{23}x_3 + \dots + a_{2n}x_n, \\ \dots, \\ \frac{dx_n}{dt} = a_{n1}x_1 + a_{n2}x_2 + \dots - \frac{1}{5}b_{nn}x_n^5 - b_{nn}\sqrt{\frac{b_{nn}k_n^3 + a_{nn}}{4}}x_n^4 - 2b_{nn}\sqrt{\frac{b_{nn}k_n^3 + a_{nn}}{4}}x_n^3 - 6b_{nn}\left(\sqrt{\frac{b_{nn}k_n^3 + a_{nn}}{4}}\right)^3x_n^2 - b_{nn}\left(k_n^3 + \frac{a_{nn}}{b_{nn}}\right)x_n. \end{cases} \quad (12)$$

The stability of system (12) is investigated by the gradient-velocity method of the Lyapunov vector function. To do this, we define the components of the gradient vector from the Lyapunov vector function  $V(x) = [(V_1(x), V_2(x), \dots, V_n(x))]$  from the equations of state (12):

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_2} = -a_{12}x_2, \quad \frac{\partial V_2(x)}{\partial x_3} = -a_{23}x_3, \dots, \quad \frac{\partial V_i(x)}{\partial x_i} = \frac{1}{5}b_{ii}x_i^5 + b_{ii}\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}x_i^4 + 2\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}x_i^3 + 6\left(\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}\right)^3x_i^2 + b_{ii}\left(k_i^3 + \frac{a_{ii}}{b_{ii}}\right)x_i, \\ \frac{\partial V_i(x)}{\partial x_{i+1}} = -a_{i,i+1}x_{i+1}, \quad i = 1, \dots, n. \end{cases} \quad (13)$$

From (12) we determine the decomposition of the components of the velocity vector by the coordinates of the system:

$$\begin{cases} \left(\frac{dx_i}{dt}\right)_{x_1} = a_{i1}x_1, \quad \left(\frac{dx_i}{dt}\right)_{x_2} = a_{i2}x_2, \dots, \\ \left(\frac{dx_i}{dt}\right)_{x_i} = -\frac{1}{5}b_{ii}x_i^5 - b_{ii}\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}x_i^4 - 2\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}x_i^3 - 6\left(\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}\right)^3x_i^2 - b_{ii}\left(k_i^3 + \frac{a_{ii}}{b_{ii}}\right)x_i, \\ \left(\frac{dx_i}{dt}\right)_{x_{i+1}} = a_{i,i+1}x_{i+1}, \dots, \quad \left(\frac{dx_i}{dt}\right)_{x_n} = a_{in}x_n, \quad i = 1, \dots, n. \end{cases} \quad (14)$$

The total time derivatives of the Lyapunov vector function are calculated as the scalar product of the gradient vector (15) by the velocity vector (14):

$$\begin{aligned} \frac{dV(x)}{dt} &= \sum_{i=1}^n \left( \sum_{j=1}^n \frac{\partial V_i(x)}{\partial x_j} \left(\frac{dx_j}{dt}\right) \right) = \\ &= \sum_{i=1}^n \{ -(a_{i1}x_1)^2 - (a_{i2}x_2)^2 - \dots - \\ &- \left[ \frac{1}{5}b_{ii}x_i^5 + b_{ii}\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}x_i^4 + 2\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}x_i^3 + \right. \\ &+ 6\left(\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}\right)^3x_i^2 + b_{ii}\left(k_i^3 + \frac{a_{ii}}{b_{ii}}\right)x_i \left. \right]^2 - \\ &- (a_{i,i+1}x_{i+1})^2 - \dots - (a_{in}x_n)^2 \}, \quad i = 1, \dots, n. \end{aligned} \quad (15)$$

It is obvious from (15) that the total time derivative of the Lyapunov vector function is a sign-negative function. Using the gradient of the Lyapunov vector function (13), we construct the Lyapunov vector function in scalar form:

$$\begin{aligned} V(x) &= \sum_{i=1}^n \frac{1}{30}b_{ii}x_i^6 + \frac{1}{5}b_{ii}\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}x_i^5 + \frac{1}{2}b_{ii} \times \\ &\times \sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}x_i^4 + 2b_{ii}\left(\sqrt{\frac{b_{ii}k_i^3 + a_{ii}}{4}}\right)^3x_i^3 + \frac{1}{2}b_{ii} \times \\ &\times \left(k_i^3 + \frac{a_{ii}}{b_{ii}}\right)x_i^2 - \frac{1}{2}a_{i2}x_2^2 - \frac{1}{2}a_{i3}x_3^2 - \dots - \frac{1}{2}a_{in}x_n^2. \end{aligned} \quad (16)$$

Function (16) vanishes at the origin, is a continuously differentiable function and has terms in the form of variables with odd degrees, i.e. the conditions of positive certainty are not obvious. Therefore, based on Morse's lemma from the theory of catastrophes, we represent function (16) in the form of a quadratic form:

$$\begin{aligned} V(x) &= \frac{1}{2}[(b_{11}k_1^3 + a_{11}) + a_{21} - a_{31} - \dots - a_{n1}]x_1^2 + \\ &+ \frac{1}{2}[-a_{12} + (b_{22}k_2^3 + a_{22}) - a_{32} - \dots - a_{2n}]x_2^2 + \\ &+ \dots + \frac{1}{2}[-a_{1n} - a_{2n} - a_{3n} - \dots + (b_{nn}k_n^3 + a_{nn})]x_n^2. \end{aligned} \quad (17)$$

From (17) the positive definiteness of the vector of the Lyapunov function (the conditions for the existence of the Lyapunov function) are given by the inequalities:

$$\begin{cases} b_{11}k_1^3 - a_{11} - a_{21} - a_{31} - \dots - a_{n1} > 0 \\ b_{22}k_2^3 - a_{12} - a_{22} - a_{32} - \dots - a_{n2} > 0 \\ b_{33}k_3^3 - a_{13} - a_{23} - a_{33} - \dots - a_{n3} > 0 \\ \dots \\ b_{nn}k_n^3 - a_{1n} - a_{2n} - a_{3n} - \dots - a_{nn} > 0. \end{cases} \quad (18)$$

Thus, the control system (3) ensures the stability of the system under any changes in uncertain parameters.

When conditions (11) are met, a steady state (4) exists and will be stable. If the system of inequalities (11) is violated, a new stationary state (5) appears and this stationary state exists and is also stable when the system of inequalities (18) is fulfilled.

4. The study of the reference model of a self-organizing adaptive control system by the gradient-velocity method of the Lyapunov vector function allows us to obtain areas of aperiodic robust stability in the form of

$$\begin{cases} -(d_1^3 + a_{21}^m + a_{31}^m + \dots + a_{n1}^m) > 0 \\ -(d_2^3 + a_{12}^m + a_{32}^m + \dots + a_{n2}^m) > 0 \\ \dots \\ -(d_n^3 + a_{1n}^m + a_{2n}^m + \dots + a_{n-1,n}^m) > 0, \end{cases} \quad (19)$$

$$\begin{cases} (d_1^3 - a_{21}^m - a_{31}^m - \dots - a_{n1}^m) > 0 \\ (d_2^3 - a_{12}^m - a_{32}^m - \dots - a_{n2}^m) > 0 \\ \dots \\ (d_n^3 - a_{1n}^m - a_{2n}^m - \dots - a_{n-1,n}^m) > 0. \end{cases} \quad (20)$$

Equating the left parts of equality (11) and (19) or (18) and (20), we obtain for the adjustable coefficients of the self-organizing adaptive control system.

$$\begin{cases} k_1^3 = (d_1^3 - a_{11})/d_{11} \\ k_2^3 = (d_2^3 - a_{22})/d_{22} \\ \dots \\ k_n^3 = (d_n^3 - a_{nn})/d_{nn}, \end{cases} \quad (21)$$

$$\text{and } \begin{cases} k_1^3 = (d_1^3 + a_{11})/d_{11} \\ k_2^3 = (d_2^3 + a_{22})/d_{22} \\ \dots \\ k_n^3 = (d_n^3 + a_{nn})/d_{nn}. \end{cases} \quad (22)$$

### Conclusion

It is now generally accepted that real control objects are nonlinear and deterministic chaos with the generation of «strange attractors» is an intrinsic property of any nonlinear deterministic dynamical system. In linearized dynamical systems, this manifests itself as a loss of robust stability.

In the conditions of uncertainty of the parameters of the control object and external influences, the actual problem of managing unstable and deterministic chaotic processes is the application of the adaptation method. In adaptive systems, external influences are compensated, and the construction of a reference model and the main control loop in the «dovetail» disaster class allows you to control unstable and deterministic chaotic processes.

The boundary conditions of the aperiodic robust stability of the reference model with the desired dynamics and the main control loop of the adaptive system allow us to solve the problem of synthesizing the main circuit of the adaptive system according to a set of quality indicators, such as stability, robustness, oscillation, speed, lack of overshoot, static accuracy, the desired type of transients in the system, etc.

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**«Қарлығаш құйрығы» апат класындағы тұрақсыз және детерминирленген хаотикалық процестер үшін адаптивті басқару жүйесінің негізгі тізбегіне шолу және синтездеу**

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**Аңдатпа.** Өткен ғасырдағы зерттеу сызықтық емес жүйелер динамикасының алуан түрлілігін анықтады және сызықтық емес динамикалық жүйелердегі ХХ ғасырдың маңызды жаңалықтарының бірі – детерминистік хаос пен оғаш тартқышқа әкелді. Басқару объектісінің параметрлері мен сыртқы әсерлердің белгісіздігі жағдайында тұрақсыз және детерминистік хаотикалық процестерді басқару үшін бейімделу әдістерін қолдану өзекті мәселе болып табылады. Сонымен қатар, қажетті динамикасы бар анықтамалық модель және адаптивті жүйенің негізгі басқару тізбегі апаттар класында құрылады, ал қажетті динамикасы бар анықтамалық модельдің аперидотты робастикалық тұрақтылығы және робастикалық тұрақтылықтың жоғары потенциалы бар адаптивті жүйенің негізгі басқару тізбегі градиент-жылдамдық Ляпунов функцияларының векторы әдісімен зерттеледі. Жалпыланған реттелетін объектінің аперидоттық робастикалық тұрақтылық шарттарынан реттелетін коэффициенттер есептеледі және басқару мақсатына қол жеткізіледі. «Қарлығаш құйрығы» апат класындағы  $m$ -кірістері мен  $n$ -шығыстары бар тұрақсыз және детерминирленген хаотикалық процестерді адаптивті басқару жүйесінің негізгі тізбегін синтездеу мәселесіне шешу ұсынылады.

**Кілт сөздер:** адаптивті, детерминистік хаос, қарлығаш құйрығы, синтез контуры, робастық тұрақтылық, инвариантты, градиент жүйесі, скаляр жұмыс, апат теориялары, статикалық дәлдік, теңсіздіктер жүйесі, динамикалық жүйелер.

**Обзор и синтез основного контура адаптивной системы управления неустойчивыми и детерминированными хаотическими процессами в классе катастроф «Ласточкин хвост»**

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**Аннотация.** Исследования последнего столетия выявили большое разнообразие динамики нелинейных систем и привели к одному из важнейших открытий ХХ века в нелинейных динамических системах – детерминированному хаосу и странному аттрактору. Актуальной проблемой в условиях неопределенности параметров объекта управления и внешних воздействий для управления неустойчивыми и детерминированными хаотическими процессами является применение методов адаптации. При этом эталонная модель с желаемой динамикой и основной контур управления адаптивной системы строится в классе катастроф «Ласточкин хвост», а аперидотическая робастная устойчивость эталонной модели с желаемой динамикой и основной контур управления адаптивной системы с повышенным потенциалом робастной устойчивости исследуется градиентно-скоростным методом вектора функций Ляпунова. Из условий аперидотической робастной устойчивости обобщенного настраиваемого объекта вычисляются настраиваемые коэффициенты и достигается цель управления. Предлагается решение задачи синтеза основного контура системы адаптивно-го управления неустойчивыми и детерминированными хаотическими процессами с  $m$ -входами и  $n$ -выходами в классе катастроф «Ласточкин хвост».

**Ключевые слова:** адаптивный, детерминированный хаос, ласточкин хвост, синтез контур, робастная устойчивость, инвариантный, градиентная система, скалярное произведение, теория катастроф, статическая точность, система неравенств, динамические системы.

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