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Issues of Rod System Strength with Initial Camber

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Abstract. *The work aims to study the stress-strain state of a pivotally-supported rod with variable cross-sectional stiffness under the action of central compression with bending. The problem is solved by the numerical finite difference method using a regular linear grid with a density equal to four. As a result of the study, graphical dependences of deflections on the change in the magnitude of the compressive load, correlated to the magnitude of the buckling load (Euler force) of the corresponding centrally compressed rod are obtained. The influence of the constant and variable thickness of the rod on deformation is investigated. The results obtained can be used both in scientific research in the field of building and in the design of various rod systems.*

Keywords: *compressed-curved rod, initial camber, variable bending stiffness, finite difference method, buckling load, bending moment, axial moment of inertia.*

Introduction

Currently, structures in the form of light rods and beams with a variable cross-section are widely used, due to the operation of which initial imperfections are often observed, which must be taken into account when calculating strength and stability. Such structural solutions are used in transport and in the construction industry as bearing elements of buildings and structures. These issues are considered by numerous scientists and are highly relevant.

For example, article [1] examined the effect of initial camber on the bending of a pivotally mounted cylindrical canopy shell. The analysis of the study of the stress-strain state of the flexible plate, taking into account the change in its initial camber beyond the elastic limit of the material in the presence of a stiffening rib, is devoted to the work [2]. Values of plastic deformations are obtained, which are comparable in weight to the thickness of the plate itself.

In work [3], a numerical study was carried out by the finite difference method, where the initial stability equation of the compressed rod is approximated by the corresponding grid equations leading the initial problem to the known problem of eigen numerals. The task of determining the critical forces of the rods of variable cross-section using the finite difference method, but without taking into account the initial camber, is considered in Article [4]. In article [5], a theoretical and experimental study of the stability of the flat bend shape of I-beams with a solid and perforated wall was carried out, taking into account variable bending stiffness and the presence of initial camber in the plane of the smallest bending stiffness; initial imperfections are defined by the eccentricity of the load application and the initial twist angle.

The above works take into account mainly the factors of operation of structures individually (or variable thickness, or in the presence of initial imperfections), this article considers the problem of the joint action of these factors, as well as in their combination.

Methods and Results

The governing differential equation (Figure 1) for calculating deflections of a compressed-bent rod has the form [6]:

$$y'' + \frac{P}{EI}y = -\frac{Pf_0(x)}{EI}, \tag{1}$$

P is the concentrated force, EI is the stiffness of the rod, $f_0(x)$ is the deflection function determining the initial (given) cambers along the axis of the rod.

Rod of constant thickness

For the i-th node (Figure 2), equation (1) will take the form [7, 8]:

$$(y_k - 2y_i + y_l) + \frac{P\lambda^2}{EI}y_i = -\frac{P\lambda^2}{EI}f_0(x). \tag{2}$$

Next, the following designations apply:

$$k = \frac{P\lambda^2}{EI}, \tag{3}$$

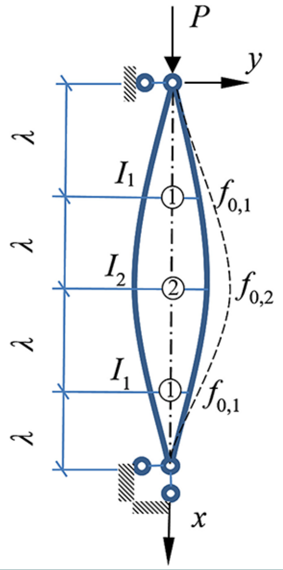

Figure 1 – Design diagram of compressed rod

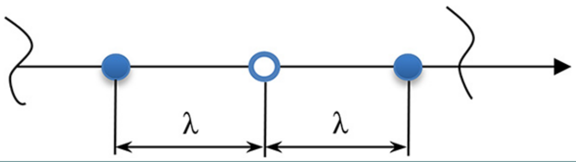

Figure 2 – A fragment of a «linear» grid

where k is the axial concentrated force parameter P.

Then equation (2) taking into account (3) will take the form:

$$y_i(-2+k_i)+(y_k+y_l)=-k_i f_0(x), \qquad (4)$$

where $k_i=\gamma_i k$, $\gamma_i=0{,}1; 0{,}2; 0{,}3; 0{,}4; 0{,}5; 0{,}6; 0{,}7; 0{,}8; 0{,}9$, $k_i=0{,}586$ is the critical parameter value for hinged rod.

Further, we write the equation (4) for the calculated grid nodes ($i=1, 2$) (Figure 1):

$$y_1(-2+0.586\gamma)+y_2=-0.586\gamma f_{0,1}, \qquad (5)$$

$$2y_1+(-2+0.586\gamma)y_2=-0.586\gamma f_{0,2}. \qquad (6)$$

Then we reduce equations (5,6) into a system of linear algebraic equations ($f_{0,1}=0.75$ cm; $f_{0,2}=1.0$ cm):

$$\begin{cases}(-2+0.586\gamma)y_1+y_2=-0.44\gamma,\\ 2y_1+(-2+0.586\gamma)y_2=-0.586\gamma.\end{cases} \qquad (7)$$

After that, perform calculations as per (7), changing the value of the γ coefficient from 0.1 to 0.9, and record the results in Table 1.

Rod of variable thickness

In this case, the initial equations (1,2) are written taking into account the variable bending stiffness $EI_i=EI(x)$, ($I_1=I_0$; $I_2=0.75I_0$)

$$(y_k-2y_i+y_l)+\frac{P\lambda^2}{EI_i}y_i=-\frac{P\lambda^2}{EI_i}f_0(x),$$

$$y_i(-2+k_i^*)+(y_k+y_l)=-k_i^* f_0(x),$$

$$k_i^*=\gamma_i k^*, \quad k^*=\frac{P\lambda^2}{EI_i},$$

where $k^*=0{,}54$ based on the stability of the rod of variable thickness, which varies according to the quadratic law.

Then, instead of (7), we get the following system of equations:

$$\begin{cases}(-2+0.54\gamma)y_1+y_2=-0.54\gamma,\\ 2y_1+(-2+0.54\gamma)y_2=-0.54\gamma.\end{cases}$$

Changing the γ parameter in the range from 0.1 to 0.9, record the results in Table 1.

Next, we plot the dependence $y_i=f(P)$ (Figure 3).

Definition of bending moments

Next, we calculate the bending moments in the calculated grid nodes (Figure 1).

In finite differences for the i^{th} grid node (Figure 2),

Table 1 – Deflections along the axis of the rod in nodes 1 and 2

	γ	y_1, cm	y_2, cm
rod of constant thickness	0.1	0.0814	0.1141
	0.2	0.1831	0.2567
	0.4	0.4879	0.6854
	0.6	1.097	1.5443
	0.8	2.002	2.713
	0.9	6.587	9.304
rod of variable thickness	0.1	0.0890	0.1192
	0.2	0.1977	0.2661
	0.4	0.5085	0.691
	0.6	1.0717	1.4722
	0.8	2.419	3.361
	0.9	4.1814	5.845

the bending moments M_i are determined through the values of nodal deflections y_i (Table 1) by the formula:

$$M_{x_i} = -EI_i \frac{d^2 y}{dx^2} = -EI_i \frac{(y_k - 2y_i + y_l)}{\lambda^2}.$$

When $\lambda = 0{,}25l$, we get

$$M_{x_i} = -\frac{16EI_i}{l^2}(y_k - 2y_i + y_l). \qquad (8)$$

The results of the calculation of bending moments according to the formula (8) with variable values of γ for various types of rods are shown in Table 2, and plots of bending moments are drawn (Figure 4).

Cross-section selection

Next, we determine the dimensions of the transverse square section (Figure 5) of a steel rod with a constant thickness (Figure 4, a) from the compressive strength condition ($\sigma_{adm} = 160$ MPa, $l = 6$ m):

$$W_x^{req} \geq \frac{M_{\max}}{\sigma_{adm}}. \qquad (9)$$

Substituting the known values, we find a:

$$\frac{a^3}{6} = \frac{86.944 \cdot 10^{-2}(36)}{2 \cdot 10^{11} \dfrac{a^4}{12} \cdot 160 \cdot 10^6}; \quad a = 0{,}37 \text{ m}.$$

Conclusions

1. In this paper, a study of the compressed-curved stress state of a hinge-supported rod with a variable thickness along its length in the presence of initial camber caused by inaccuracy of manufacture or deformation during operation of the rod, under

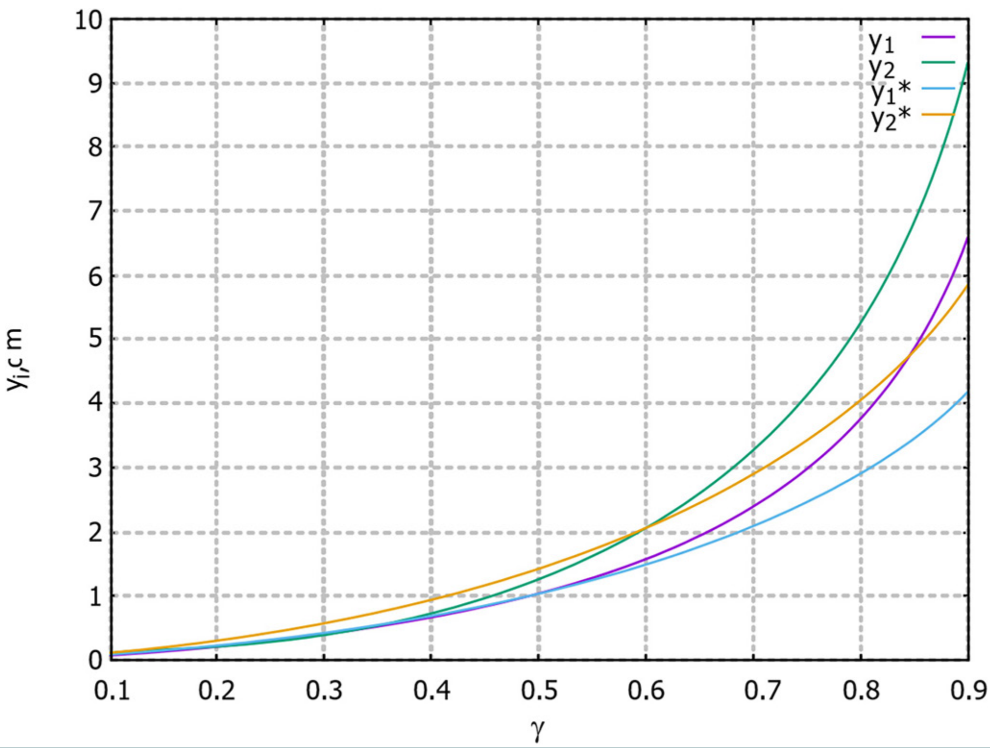

Figure 3 – Dependence of deflections y_i along rod axis on the value of axial load P (y_1, y_2 for constant thickness rod, y_1^*, y_2^* for variable thickness rod)

Table 2 – Ordinates of plots M_{x_i}

	$\frac{M_{x_i} l^2}{EI_0}$											
	γ = 0,1		γ = 0,2		γ = 0,4		γ = 0,6		γ = 0,8		γ = 0,9	
nodes	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**
rod of constant thickness	0,7792	1,0464	1,752	2,3552	4,6464	6,32	10,3952	14,3136	20,656	22,752	61,92	86,944
rod of variable thickness	0,9408	0,9668	2,0688	2,1888	5,216	5,840	10,7392	12,816	24,592	28,224	40,2848	53,2352

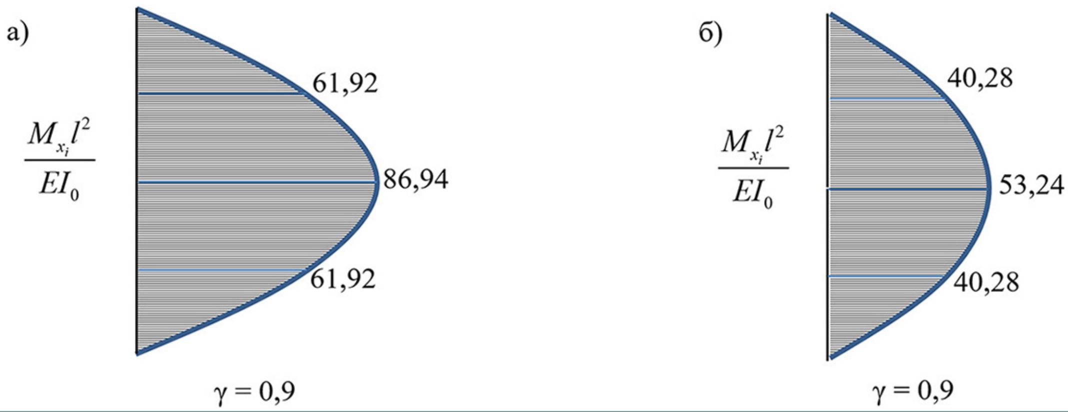

Figure 4 – Diagrams of bending moments along the length of the rod: a) constant thickness, b) variable thickness

the action of axial compressive force P (Figure 1).

2. On the basis of the governing differential equation of the second order, resolving finite-difference equations are obtained, taking into account the variable bending stiffness and the presence of an initial camber (displacements of the calculated nodes along the length of the rod).

3. The change in the values of displacements y_1 and y_2 of a rod of constant and variable thickness from a change in the coefficient ($\gamma = P/P_{cr}$) is studied (Figure 3, Table 1). At the same time, an increase in the value of γ leads to a smooth (exponentially) increase in y_i; with the same values of γ, the deflections of y_i for a rod of variable thickness are less than for a rod of constant thickness.

4. The theoretical provisions and applied results presented in this work can be used in scientific research in the field of construction and in the design of elements of buildings and structures for various purposes.

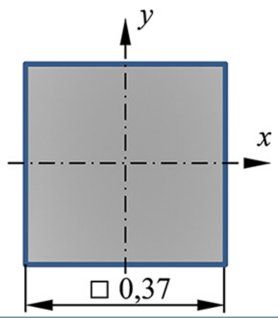

Figure 5 – Cross section of a rod with constant bending stiffness

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Бастапқы иілім сырық жүйесінің беріктігі мәселелері

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Аңдатпа. *Жұмыстың мақсаты – иілісі бар орталық қысу әсерінен ауыспалы қатаңдығы бар топсалы-тірелген сырықтың кернеулі-деформацияланған күйін зерттеу. Қойылған міндет төртке тең тұрақты сызықтық торды қолдана отырып, шекті айырмашылық сандық әдісімен шешілді. Зерттеу нәтижесінде ауытқулардың тиісті орталық сығылған сырықтың дағдарыс күшінің (Эйлер күшінің) шамасына байланысты сығылатын жүктеме шамасының өзгеруіне графикалық тәуелділігі алынды. Тұрақты және ауыспалы сырық қалыңдығының деформациясына әсері зерттелді. Алынған нәтижелер құрылыс саласындағы ғылыми зерттеулерде де, әртүрлі сырық жүйелерін жобалау кезінде де қолданыла алады.*

Кілт сөздер: *сығылған иілген сырық, бастапқы иілу, өзгермелі иілу қатаңдығы, шекті айырмашылық әдісі, дағдарыс күші, иілу моменті, өстік инерция моменті.*

Вопросы прочности стержневой системы с начальной погибью

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Аннотация. *Целью работы является исследование напряженно-деформированного состояния центрального сжатия с последующим изгибом шарнирно-опертого стержня с переменной изгибной жесткостью по его длине. Поставленная задача решена численным методом конечных разностей с применением регулярной линейной сетки густотой, равной четырем. В результате исследования получены графические зависимости прогибов от изменения величин сжимающей нагрузки, соотнесенной к величине критической силы (силы Эйлера) соответствующего центрально-сжатого стержня. Исследовано влияние на деформацию постоянной и переменной толщины стержня. Полученные результаты могут найти применение как в научных изысканиях в области строительства, транспорта, так и при проектировании различных стержневых систем.*

Ключевые слова: *сжато-изогнутый стержень, начальная погибь, переменная изгибная жесткость, метод конечных разностей, критическая сила, изгибающий момент, осевой момент инерции.*

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