DOI 10.52209/1609-1825_2022_1_160 UDC 624.04

Dynamic Calculation of the Turbo Units' Frame Foundation Based on One-Dimensional Rod Model

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Abstract. *The issues of dynamic calculation of frame reinforced concrete foundations of turbine generators with different rotor speed are presented in this work. The calculations were carried out by the well-known force method, while, to simplify the calculation procedure, it is proposed to go from the original three-dimensional problem to the equivalent bar (one-dimensional) model. The original system is considered as a dynamic task with two degrees of free place (for two concentrated point masses) under the action of the horizontally concentrated vibration load caused by the rotation of the turbine rotor. The system was calculated for free and forced vibrations without taking into account damping based on the decomposition of vibrations into their main forms. The characteristics of free and forced vibrations are obtained, an analytical assessment of vibration safety is made. The influence of the number of revolutions of the rotor on the value of the root-mean-square speed of forced vibrations of the structure is investigated. The results are obtained for the scientific interest and can be applied in design practice.*

Keywords: *vibration, vibration safety, reinforced concrete foundation, vibrations, dynamic calculation, root-mean-square speed, rotor, stator, turbine, free vibrations, forced vibrations.*

Introduction

In thermal power engineering, the machines with nominally balanced moveable parts (turbine units, turbine generators) are widely used, the work of which during operation creates a dynamic effect on their supporting devices and foundation, i.e. on the building structures of the construction.

To assess the vibration safety of such machines and their foundations, it is necessary to perform appropriate dynamic calculations in order to determine dynamic displacements, deformations, stresses. The complexity of calculations in these cases is due to the fact that the objects of study (machine foundations) are spatial structures with a multiple-degree freedom.

In this work, in order to perform applied engineering calculations of the frame foundations of turbo units, it is proposed to reduce the initial three-dimensional problem of mechanics of a deformable solid body to a rod (one-dimensional) problem. At the same time, only bending deformations are taken into account, i.e. torsional deformations with their relatively small effects on the operation of the structure are not taken into account.

Let us consider the calculation procedure of frame reinforced concrete foundations for standard turbine units with the number of rotations of moveable parts $n^*=3000$ rpm, the total mass of the turbine is 617 t, the weight of the rotor is 44.8 t. The design solution is schematically shown in Figure 1.

The calculation scheme of the turbine unit is presented in Figure 2, a. as a system with multiple-degree freedom.

The initial calculation data:

Frequency of forced oscillations:

$$\theta = \frac{\pi \cdot n^*}{30} = \frac{3,14 \cdot 3000}{30} = 314\,\frac{\text{rad}}{\text{s}}.$$

$P_1=145$ t. – weight of all connecting columns,
$P_2=1434$ t. – weight of bottom plate,
$P_1=354,74$ t. – weight of the upper platform,
$P=972$ t. – the total weight of the topside structure including the weight of the turbine itself.

1. Calculation for free vibrations

The canonical equations of the method of forces for free vibrations have the form [2]

$$\begin{cases} \left(\delta_{11} m_0 - \dfrac{1}{\omega_i^2}\right) y_{1,i} + \left(\delta_{12} k_1 m_0\right) y_{2,i} = 0 \\ \left(\delta_{21} m_0\right) y_{1,i} + \left(\delta_{22} k_1 m_0 - \dfrac{1}{\omega_i^2}\right) y_{2,i} = 0 \end{cases} \tag{1}$$

In Figure 2 (b, c) the unit epures from unit forces $\overline{x_1} = \overline{x_2} = 1$ are built.

The coefficients of equation (1) are calculated by the Mohr formula using the Verishchagin rule.

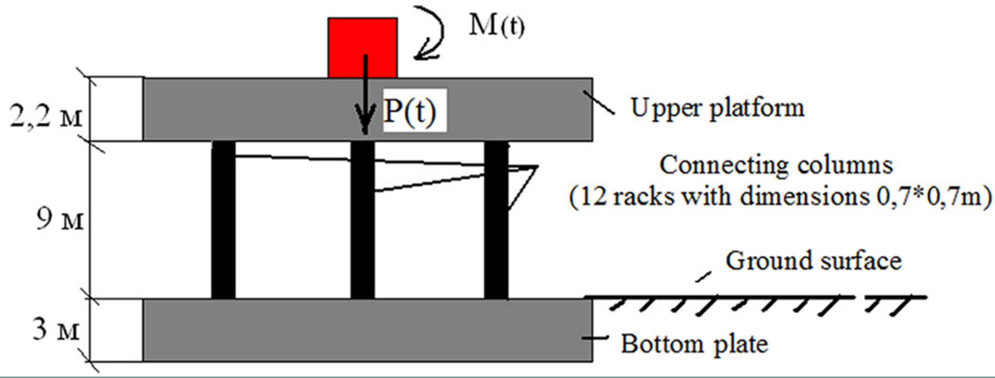
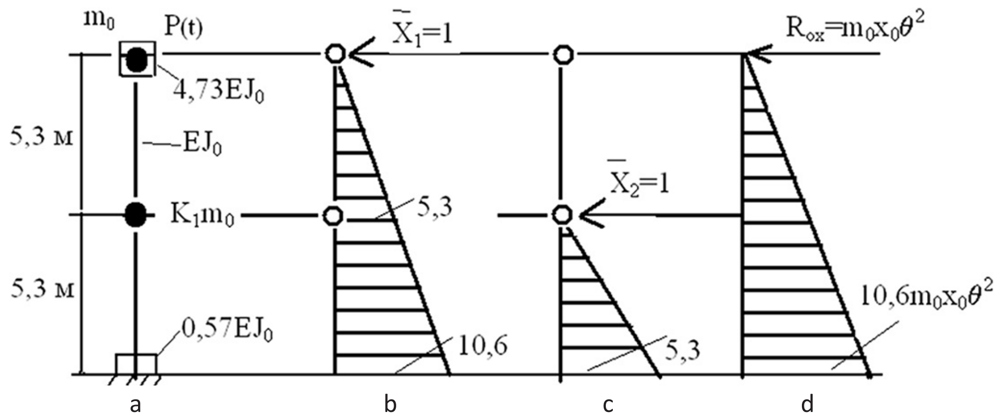


Figure 1 – Structural scheme of frame reinforced concrete foundation



a) Calculation scheme b) Epure $\overline{M_1}$ c) Epure $\overline{M_2}$ d) Epure M_p

Figure 2 – To the calculation of the frame foundation

$$\delta_{11} = \frac{1}{EJ}(\overline{M_1})\cdot(\overline{M_1}) = \frac{524,63}{EJ_0};$$
$$\delta_{12} = \delta_{21} = \frac{1}{EJ}(\overline{M_1})\cdot(\overline{M_2}) = \frac{224,33}{EJ_0}; \qquad (2)$$
$$\delta_{22} = \frac{1}{EJ}(\overline{M_2})\cdot(\overline{M_2}) = \frac{125,57}{EJ_0}.$$

Let us take the notation:

$$\lambda_i = \frac{EJ_0}{m_0\omega_i^2} - \text{frequency parameter} \qquad (3)$$

We take the value ($K_1 = 0,15$) (taking into account expressions (2.3)), we write down the equation (1)

$$\begin{cases}(524,63-\lambda_i)y_{1,i} + (33,65)\,y_{2,i} = 0\\(224,33)y_{1,i} + (18,84-\lambda_i)\,y_{2,i} = 0\end{cases} \qquad (4)$$

Using equations (4), we compose a «secular» equation to determine the frequencies of natural oscillations [3]:

$$D = \begin{vmatrix}(524,63-\lambda_i); & 33,65;\\ 224,33; & (18,84-\lambda_i);\end{vmatrix} = 0. \qquad (5)$$

Having expanded the determinant (5), we define the eigenvalues λ_1, λ_2.

$$\lambda_1 = 538,1434;\ \ \lambda_2 = 4,3370. \qquad (6)$$

Using the values (6), we construct the main modes of natural vibrations, substituting the values (6) into equation (4).

a) the 1st main form ($\lambda_1 = 538,1434$)

$$\begin{cases}(524,63-538,1434)y_{11} + (33,65)\,y_{21} = 0\\(224,33)y_{11} + (18,84-538,1434)\,y_{21} = 0\end{cases} \qquad (7)$$

From equations (7):

$$y_{21} = 1;\ \ y_{11} = 2,49; \qquad (8)$$

b) the 2nd main form ($\lambda_1 = 4,3370$)

$$\begin{cases}(524,63-4,3370)y_{12} + (33,65)\,y_{22} = 0\\(224,33)y_{12} + (18,84-4,3370)\,y_{22} = 0\end{cases} \qquad (9)$$

From equations (9):

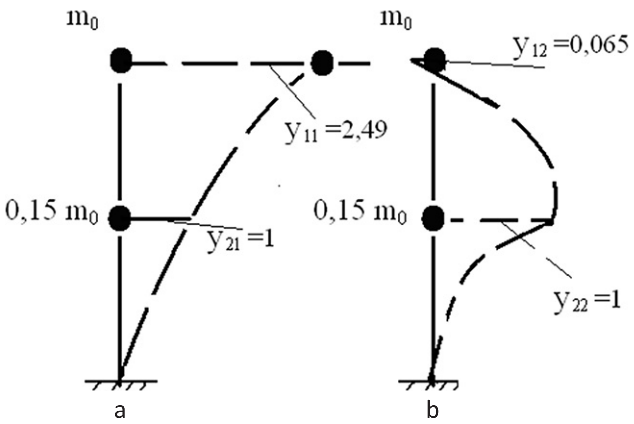
$$y_{22} = 1;\ \ y_{12} = 0,065. \qquad (10)$$

Using the values (8.10), we build the main modes of natural oscillations, Figure 3.

Using the values λ_i of the formula (3), we define the circular frequencies of natural vibrations:

$$\omega_1 = \frac{1}{\sqrt{\lambda_1}}\sqrt{\frac{EJ_0}{m_0}} = 0,0431\sqrt{\frac{EJ_0}{m_0}} - \text{fundamental period,}$$
$$\omega_2 = \frac{1}{\sqrt{\lambda_2}}\sqrt{\frac{EJ_0}{m_0}} = 0,4802\sqrt{\frac{EJ_0}{m_0}} - \text{first period.} \qquad (11)$$

We calculate the value of the actual bending stiffness EJ_0 (Figure 4).



a) 1st main form b) 2nd main form

Figure 3 – The main modes of natural vibrations

$$EJ_0 = EJ_x = EJ_{reduced}, \qquad (12)$$

where $J_{reduced}$ reduced moment of inertia of the connecting column [3]:

$$J_{прив} = 12[J_x^0 + A^0 \cdot 2,25^2];\ h = b = 0,7 \text{ m};$$

$$A_0 = h \cdot b = 0,7 \cdot 0,7 = 0,49 \text{ m};$$

$$J_{x0} = \frac{bh^3}{12} = \frac{0,7 \cdot 0,7^3}{12} = 0,02 \text{ m}^4.$$

$$J_{reduced} = 12[0{,}02 + 0{,}49(2{,}25)^2] = 30 \text{ m}^4;$$

$E = 2{,}1 \cdot 10^4$ MPa $= 2{,}1 \cdot 10^6$ t/m² – modulus of elasticity of the reinforced concrete column material;

$$EJ_0 = 2,1 \cdot 10^6 \cdot 30 = 63 \cdot 10^6 \text{ t/m}^2. \qquad (13)$$

По выражениям (11) с учетом (13) и According to the expressions (11) taking into account (13) and ($m_0 = 145\ t.$) we calculate

$\omega_1 = 0,0431\sqrt{\dfrac{63 \cdot 10^6}{145}} = 28,41\ \dfrac{\text{rad}}{\text{s}}$ – fundamental period,

$\omega_2 = 0,4802\sqrt{\dfrac{63 \cdot 10^6}{145}} = 317,71\ \dfrac{\text{rad}}{\text{s}}$ – the first period, i.e.

$$\omega_1 = 28,41\ \frac{\text{rad}}{\text{s}};\ \omega_2 = 317,71\ \frac{\text{rad}}{\text{s}}. \qquad (14)$$

2. Calculation for forced oscillations

The calculation scheme of the frame foundation as a system with two degrees of freedom (according to one-dimensional model) has the form (Figure 5).

Calculation data: $P_{рот} = 44{,}8$ t, $\rho = 0{,}75$, $\theta = 314$ s^{-1}. По [3]

$$P_{y0}(t) = P \cdot \cos(\theta \cdot t) = 44,8 \cos(\theta \cdot t);$$

$$M_{z,0}(t) = P \cdot \rho \cdot \cos(\theta \cdot t) = 44,8 \cdot 0,7 \cdot \cos(\theta \cdot t) =$$
$$= 33,6 \cdot \cos(\theta \cdot t).$$

At $\cos(\theta \cdot t) = 1$: calculate the horizontal component of the vibration load:

$$P_y = P_{y,0}^{\max}(t) + M_{y,0}^{\max}(t) = 44,8 + \frac{33,6}{10,6} = 48 \text{ t}.$$

According to [1], we calculate the second variant of the vibration load

$$P_g = k \cdot \eta \cdot P^{н}, \qquad (15)$$

where $k = 4$ – overload coefficient [1];

$\eta = 2$ – dynamic coefficient [1];

$$P^{н} = 0,2 \cdot P_{рот} = 0,2 \cdot 44,8 = 8,96 \text{ t}.$$

According to (15) $P_g = 4 \cdot 2 \cdot 8{,}96$ t.

We calculate the average (reduced) value of the bending stiffness (Figure 2), according to the dimensions of the structural solution of the elements of the frame foundation (Figure 4)

$$EJ_{ср} = EJ_0 \cdot \frac{(2,2 \cdot 4,73 + 1 \cdot 8 + 0,57 \cdot 1,5)}{2,2 + 8 + 1,5} = 1,6462 EJ_0.$$

We calculate the greatest static deflection from force $P = 72$ t. (Figure 5 b, c)

$$y_{\max} = \frac{1}{EJ_{ср}} \cdot (M_p) \cdot (\overline{M}_1) = \frac{1}{1,6462 \cdot 63 \cdot 10^6} \times$$
$$\times \left(\frac{1}{2} \cdot 763,2 \cdot 10,6 \cdot \frac{2}{3} \cdot 10,6\right) = 0,276 \cdot 10^{-3} \text{ m};$$

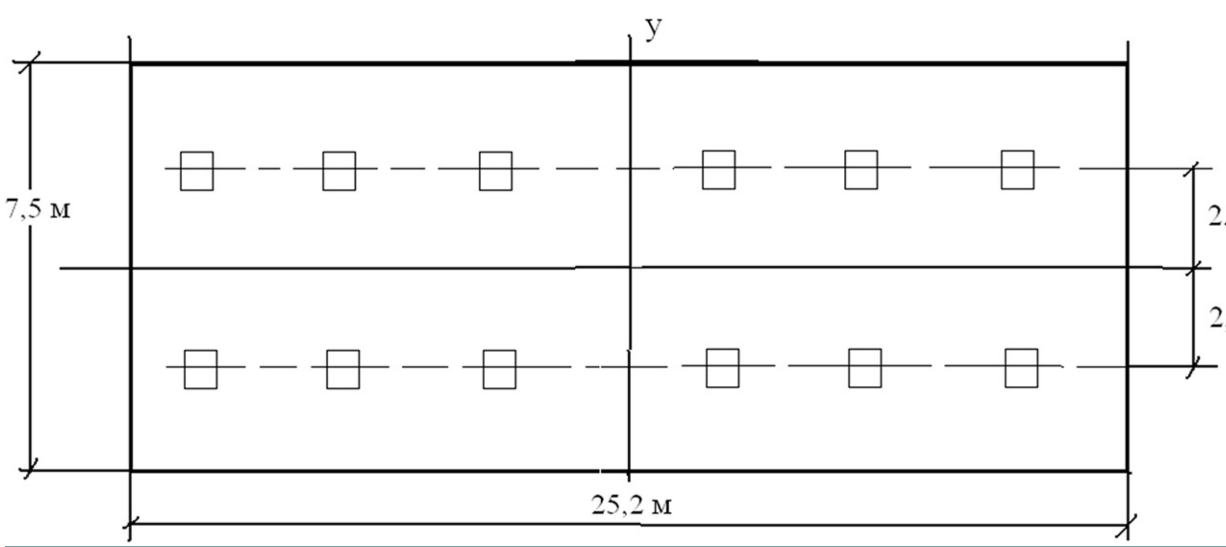
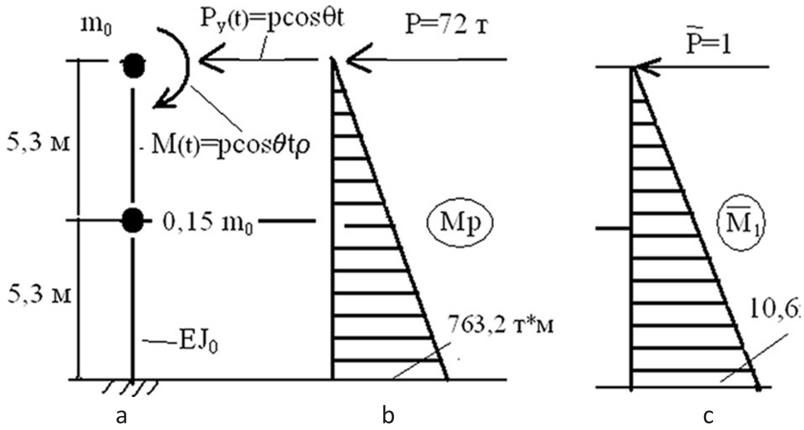


Figure 4 – Plan of the connecting columns of the upper structure of the frame foundation



a) Calculation scheme of the rod b) Load epure of moments c) Unit epure of moments

Figure 5 – To the calculation for forced oscillations

$$y_{\max} = 0,276 \cdot 10^{-3}\ \text{m} = 276\ \mu\text{m}.$$

We calculate the root-mean-square speed of vibrational oscillations [1]

$$V_{\text{ср,кв}} = \sqrt{\sum_{i=1}^{n} A_i^2 \frac{\omega_i^2}{2}}\ \frac{\text{m}}{\text{s}}, \tag{16}$$

where A_i – amplitude of the i-th harmonic oscillation, ω_i – the circular frequency of the i-th form of natural vibrations (expression 14).

According to [1], we calculate the amplitudes

$$A_i = \frac{P_0}{m \cdot (\theta^2 - \omega_i^2)}. \tag{17}$$

According to (17):

$$A_1 = \frac{44,8}{61,7 \cdot (314^2 - 28,41^2)} = 0,742 \cdot 10^{-5}\ \text{m};$$

$$A_2 = \frac{44,8}{61,7 \cdot (317,71^2 - 314^2)} = 30,91 \cdot 10^{-5}\ \text{m};$$

According to (16):

$$V_{\text{ср,кв}} = \\ = \sqrt{\frac{(0,742 \cdot 10^{-5})^2 \cdot (28,41)^2 + (30,91 \cdot 10^{-5})^2 \cdot (317,71)^2}{2}} = \\ = 6,86 \cdot 10^{-2}\ \frac{\text{m}}{\text{s}};$$

$$V_{\text{ср,кв}} = 6,86 \cdot 10^{-2}\ \frac{\text{m}}{\text{s}} = 0,686\ \frac{\text{mm}}{\text{s}} \prec V_{\text{ср,кв}} = 1,18\ \frac{\text{mm}}{\text{s}};$$

standard value [2]. Thus, the technical condition (vibration safety) of the frame foundation is assessed as «good».

Table 1 shows the values of vibration amplitudes and root-mean-square velocities depending on the change in the rotor speed n^*.

According to Table 1, it can be seen that with an increase in the number of rotor revolutions, A_1 amplitude decreases, while A_2 amplitude increases. At $n^* > 2500$ rpm, A_2 amplitude increases sharply, while the root-mean-square oscillation speed monotonically increases. Based on the results of Table 1, the graphs have been built (Figure 6):

$$A_1 = f_1(n^*);\ A_2 = f_2(n^*);\ V_{root\text{-}meam\text{-}square} = f_3(n^*).$$

Table 1

№	n*, rpm	θ, rad/s	A_1	A_2	$V_{root\text{-}mean\text{-}square}$, m/s
1	500	52,33	37,59*10^{-5}	0,72*10^{-5}	0,771*10^{-2}
2	1000	104,66	7,17*10^{-5}	0,81*10^{-5}	0,23*10^{-2}
3	1500	156,99	3,05*10^{-5}	0,95*10^{-5}	0,214*10^{-2}
4	2000	209,34	1,69*10^{-5}	1,28*10^{-5}	0,286*10^{-2}
5	2500	261,68	1,07*10^{-5}	2,24*10^{-5}	0,498*10^{-2}
6	3000	314	0,742*10^{-5}	30,91*10^{-5}	6,86*10^{-2}

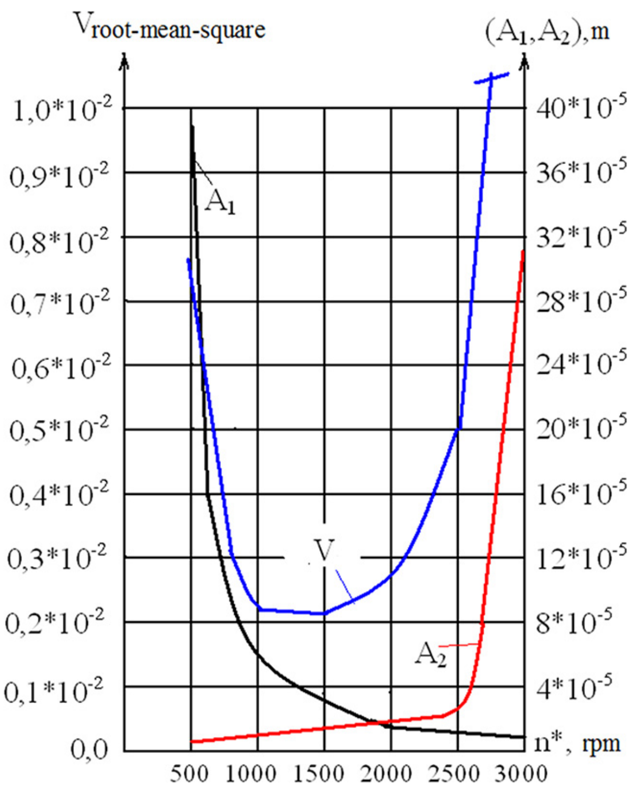


Figure 6 – Dependence of the amplitude of oscillations and root-mean-square speed on the number of the rotor rotations

Conclusion

1. In this work, instead of the generally accepted calculation of reinforced concrete frame foundations of turbines as a spatial (three-dimensional) model, it is proposed to calculate them using a simplified one-dimensional rod system (Figure 2a).

2. On the basis of the force method, a direct dynamic calculation of a rod system with two-degree freedom was carried out: a) calculation for free vibrations in the form of «standing» waves (Figure 3); b) calculation for forced vibrations on the action of vibrational harmonic load created by the rotating part of the turbine (rotor). Dynamic displacements (vibration amplitudes), mean-square speeds of vibration vibrations are calculated.

3. A method for assessing vibration during operation of turbine foundations is presented (based on [3]): it has been established that when the rotor speed $n^* \leq 3000$ rpm, the technical condition of the turbine foundation is «good».

4. The dependence of the dynamic characteristics of the turbine frame foundation (amplitude and root-mean-square speed of forced oscillations depending on the rotor speed cycle n^* (table 1, Figure 6)) has been investigated. It was found that with an increase in the number of rotor rotations, the following occurs: the amplitude A_1 decreases monotonically; the amplitude A_1 increases monotonically, while for $n^* > 2500$ rpm – it increases sharply; the value of the root-mean-square oscillation speed at the beginning «falls» at ($n^* = 500 - 1500$ rpm), and then increases sharply (at $n^* > 1500$ rpm); at the same time, for $n^* = 3000$ rpm, it increases significantly.

5. The theoretical and practical results obtained in this work will find application both in scientific research and practical construction design of reinforced concrete frame foundations, as well as in assessing their vibration safety. This study has shown that the calculation according to the one-dimensional (rod) model significantly reduces the volume and complexity of the computational operations performed.

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Бір өлшемді өзек моделіне негізделген турбоагрегаттардың рамалық іргетасын динамикалық есептеу

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Аңдатпа. *Жұмыста ротор айналымының әр түрлі саны бар турбогенераторлардың рамалық темірбетон негіздерін динамикалық есептеу мәселелері қарастырылады. Есептеулер белгілі күштер әдісімен жүзеге асырылады, ал есептеу процедурасын жеңілдету үшін бастапқы үш өлшемді тапсырмадан эквивалентті өзек (бір өлшемді) моделіне өту ұсынылады. Бастапқы жүйе турбина роторының айналуынан туындаған көлденең бағытталған діріл жүктемесінің әсерінен екі еркіндік дәрежесі бар динамикалық тапсырма ретінде қарастырылады (екі шоғырланған нүктелік масса үшін қаралыстырылады). Тербелістердің олардың басты нысандарына ыдырауының негізінде демпферлеуді (өшуді) есепке алмағанда еркін және мәжбүрлі тербелістерге арналған жүйені есептеу орындалды. Еркін және мәжбүрлі тербелістердің сипаттамалары алынды, діріл қауіпсіздігін аналитикалық бағалау жүргізілді. Ротордың айналу санының құрылымның мәжбүрлі тербелістерінің орташа квадраттық жылдамдығының мәніне әсері зерттелді. Алынған нәтижелер ғылыми қызығушылық тудырады және оларды жобалау тәжірибесінде қолдануға болады.*

Кілт сөздер: *тербелуі, темірбетон іргетасы, тербеліс, динамикалық есептеу, ротор, статор, турбина, еркін тербелістер, еріксіз тербелістер.*

Динамический расчет рамного фундамента турбоагрегатов на основе одномерной стержневой модели

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Аннотация. *В работе рассматриваются вопросы динамического расчета рамных железобетонных фундаментов турбогенераторов с различным числом оборотов ротора. Расчеты выполнены известным методом* *сил, при этом для упрощения процедуры расчета предлагается от исходной трехмерной задачи перейти к эквивалентной стержневой (одномерной) модели. Исходная система рассматривается как динамическая задача с двумя степенями свободы (для двух сосредоточенных точечных масс) при действии на нее горизонтально сосредоточенной вибрационной нагрузки, вызванной вращением ротора турбины. Выполнен расчет системы на свободные и вынужденные колебания без учета демпферирования (затухания) на основе разложения колебаний на главные их формы. Получены характеристики свободных и вынужденных колебаний, выполнена аналитическая оценка вибрационной безопасности. Исследовано влияние числа оборотов ротора на величину среднеквадратичной скорости вынужденных колебаний сооружения. Полученные результаты представляют научный интерес и могут быть применены в практике проектирования.*

Ключевые слова: *вибрация, вибробезопасность, железобетонный фундамент, колебания, динамический расчет, среднеквадратичная скорость, ротор, статор, турбина, свободные колебания, вынужденные колебания.*

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